Soluzioni 11/11/2020

1

$$\begin{cases} P(X>6)=0.1 \\ P(X<3)=0.05 \end{cases}$$

$$\begin{cases} P\left(\frac{X-\mu}{\sigma}>\frac{6-\mu}{\sigma}\right)=0.1 \\ P\left(\frac{X-\mu}{\sigma}<\frac{3-\mu}{\sigma}\right)=0.05 \end{cases}$$

$$\begin{cases} P\left(Z>\frac{6-\mu}{\sigma}\right)=0.1 \\ P\left(Z>-\frac{3-\mu}{\sigma}\right)=0.05 \end{cases}$$

$$\begin{cases} \dfrac{6-\mu}{\sigma} = 1.282 \\ -\dfrac{3-\mu}{\sigma} = 1.645 \end{cases}$$

$$\begin{cases} 6-\mu = 1.282\,\sigma \\ -3+\mu = 1.645\,\sigma \end{cases}$$

$$3 = 2.927\,\sigma$$

$$\sigma = \frac{3}{2.927} = 1.025$$

$$\mu = 3 + 1.645 \times 1.025 =$$

$$= 3 + 1.686 =$$

$$= 4.686$$

$$\mu = 4.686$$

$$\sigma = 1.025$$

2

$\lambda = 12 / \text{ore}$ $\qquad \frac{60}{12} = 5$

primi $5'$ $\qquad \lambda = 1$

$$P(X > 2) = 1 - P(X \leq 2) =$$

$$= 1 - P(X=0) - P(X=1) - P(X=2) =$$

$$= 1 - e^{-\lambda} - e^{-\lambda}\lambda - e^{-\lambda}\frac{\lambda^2}{2} =$$

$$= 1 - e^{-1}\left(1 + 1 + \frac{1}{2}\right) = 1 - \frac{5}{2e} \approx 0.08$$

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(3) È un'ipergeometrica con

$r = 10 \qquad b = 20 \qquad N = 30$

$n = 5 \qquad k = 3$

$$P(X=3) = \frac{\binom{10}{3}\binom{20}{2}}{\binom{30}{5}} =$$

$$= \frac{\overset{\cancel{10}\cdot\cancel{9}\cdot 8}{\cancel{10!}}\ \overset{\overset{\cancel{10}\ 5}{\cancel{20}}\cdot 19}{\cancel{20!}}\ \overset{5\cdot\cancel{4}}{\cancel{5!}}\ \cancel{25!}}{\underset{\underset{3}{\cancel{30}}\cdot 29\cdot \underset{7}{\cancel{28}}\cdot \underset{3}{\cancel{27}}\cdot \underset{13}{\cancel{26}}}{\cancel{3!}\ \cancel{7!}\ \cancel{2!}\ \cancel{18!}\ \cancel{30!}}} =$$

$$= \frac{8\cdot 5\cdot 19\cdot 5}{3\cdot 29\cdot 7\cdot 3\cdot 13} \simeq 0.16$$

(4) $V = (R + R_0) I$

$I = 10^{-2}$

$R_0 = 10^3$

$R \sim U(a, b)$

$a = 9 \times 10^2$

$b = 11 \times 10^2$

$$m_V = E[V] = I\left(E[R] + R_0\right) =$$

$$= I\left(\frac{a+b}{2} + R_0\right) = 10^{-2}\left(10^3 + 10^3\right) = 20 \text{ mV}$$

$$Var(R) = \frac{1}{12}(b-a)^2 = \frac{10^4}{3}(mA)^2$$

$$\sigma_V^2 = Var(V) = Var\left(I(R+R_0)\right) =$$

$$= I^2 \, Var(R) = I^2 \frac{(b-a)^2}{12} =$$

$$= 10^{-4} \frac{(2 \times 10^2)^2}{12} = \frac{1}{3}(mV)^2$$

$$\mathrm{Cov}(R,V) = \mathrm{Cov}\big(R, I(R+R_0)\big) =$$

$$= I\left\{\mathrm{Cov}(R,R) + \underbrace{\cancel{\mathrm{Cov}(R,R_0)}}_{=0}\right\} =$$

$$= I\,\mathrm{Var}(R)$$

$$\rho = \frac{\mathrm{Cov}(R,V)}{\sqrt{\mathrm{Var}(R)\,\mathrm{Var}(V)}} = \frac{I\cdot\mathrm{Var}(R)}{\sqrt{I^2\,\mathrm{Var}(R)^2}} = 1$$