Consider the equation

$$\ddot{c}_n + \omega_n^2\, c_n = \frac{\Psi_n}{\omega_n^2} \tag{1}$$

where $\omega_n^2 = \lambda_n^4$ and

$$\Psi_n = -2\,\beta^2\,\left\{a\left[\varphi_n''(x_k + a) + \varphi_n''(x_k - a)\right] - \left[\varphi_n'(x_k + a) - \varphi_n'(x_k - a)\right]\right\}. \tag{2}$$

In the case of a simply-supported beam we have $\varphi_n(x) = \sqrt{2}\,\sin(n\pi x)$, $\omega_n = (n\pi)^2$ and

$$\frac{\Psi_n}{\omega_n^2} = 4\,\sqrt{2}\,\beta^2\,\frac{a\,n\,\pi\,\cos(a\,n\,\pi) - \sin(a\,n\,\pi)}{n^3\,\pi^3}\,\sin(n\pi x_k) \tag{3}$$

If a is constant, equations (1)-(3) can be solved in closed form. Let

$$w(x,t) = \sum_{n=0}^{\infty} c_n(t)\,\varphi_n(x) \tag{4}$$

be the solution for the profile function.

The motion of the ball is given by

$$\ddot{z} = \frac{1}{\varepsilon}\,f = \frac{\beta^2}{\varepsilon}\,a^3 \tag{5}$$

with initial conditions

$$z(0) = w(x_k, t_k); \quad \dot{z}(0) = v_z \tag{6}$$

From the model equations we also have

$$a = a(t) = \gamma\,\sqrt{w(x_k,t) - z(t)} \tag{7}$$

with

$$\gamma = \sqrt{\frac{L^3\,R}{J}}.$$

From equation (7) we have

$$w(x_k, t) = z(t) + \left(\frac{a}{\gamma}\right)^2 \equiv u(t) \tag{8}$$

In principle, if the equations were solved simultaneously with $a(t)$, the expression $u(t)$ in equation (8) would be equal to $w(x_k,t)$ given by (4). Since equations (1)-(3) are solved under the assumption of constant a, they will differ. We may try two possible approaches:

1. find a (constant) which minimizes

$$\int_0^{t_f} (u(t) - w(x_k,t))^2 dt \tag{9}$$

   where t_f is the final time of impact;

2. an updated constant value of a can be obtained from equation (8) and plugged back into equations (1)-(3) for an iterative procedure.