

**(\*\* SISTEMI LINEARI \*\*)**

**(\* PIVOT PARZIALE \*)**

**(\* PRIMO ESEMPIO (Dal libro) \*)**

**Clear[a];**

**a = {{0.003, 59.14}, {5.291, -6.13}};**

**x = {x1, x2};**

**b = {59.17, 46.78};**

**ab = {Join[a[[1]], {b[[1]]}], Join[a[[2]], {b[[2]]}};**

**MatrixForm[a]**

**MatrixForm[b]**

**MatrixForm[x]**

**MatrixForm[ab]**

**sys = Dot[a, x] == b**

**Solve[sys, x]**

$$\begin{pmatrix} 0.003 & 59.14 \\ 5.291 & -6.13 \end{pmatrix}$$

$$\begin{pmatrix} 59.17 \\ 46.78 \end{pmatrix}$$

$$\begin{pmatrix} x1 \\ x2 \end{pmatrix}$$

$$\begin{pmatrix} 0.003 & 59.14 & 59.17 \\ 5.291 & -6.13 & 46.78 \end{pmatrix}$$

$$\{0.003 x1 + 59.14 x2, 5.291 x1 - 6.13 x2\} = \{59.17, 46.78\}$$

$$\{x1 \rightarrow 10., x2 \rightarrow 1.\}$$

**ab1 = {ab[[1]], ab[[2]] - ab[[1]] \* (ab[[2]][[1]] / ab[[1]][[1]])};**

**MatrixForm[ab1]**

$$\begin{pmatrix} 0.003 & 59.14 & 59.17 \\ 0. & -104309. & -104309. \end{pmatrix}$$

**x2sol = ab1[[2]][[3]] / ab1[[2]][[2]]**

**x1sol = (ab1[[1]][[3]] - ab1[[1]][[2]] \* x2sol) / ab1[[1]][[1]]**

**1.**

**10.**

**(\* SECONDO ESEMPIO \*)**

```

Clear[a, ab, ab1, b]
eps = 1.0 * 10^(-12);
a = {(eps, 2.1), {5.2, -3.1}};
b = {a[[1]][[1]] + 10 * a[[1]][[2]], a[[2]][[1]] + 10 * a[[2]][[2]]};
ab = {Join[a[[1]], {b[[1]]}], Join[a[[2]], {b[[2]]}]};
MatrixForm[a]
MatrixForm[b]
MatrixForm[ab]
sys = Dot[a, x] == b
Solve[sys, x]


$$\begin{pmatrix} 1. \times 10^{-12} & 2.1 \\ 5.2 & -3.1 \end{pmatrix}$$



$$\begin{pmatrix} 21. \\ -25.8 \end{pmatrix}$$



$$\begin{pmatrix} 1. \times 10^{-12} & 2.1 & 21. \\ 5.2 & -3.1 & -25.8 \end{pmatrix}$$



$$\{1. \times 10^{-12} x_1 + 2.1 x_2, 5.2 x_1 - 3.1 x_2\} = \{21., -25.8\}$$



$$\{x_1 \rightarrow 1., x_2 \rightarrow 10.\}$$


ab1 = {ab[[1]], ab[[2]] - ab[[1]] * (ab[[2]][[1]] / ab[[1]][[1]])};
MatrixForm[ab1]
x2sol = ab1[[2]][[3]] / ab1[[2]][[2]]
x1sol = (ab1[[1]][[3]] - ab1[[1]][[2]] * x2sol) / ab1[[1]][[1]]


$$\begin{pmatrix} 1. \times 10^{-12} & 2.1 & 21. \\ 0. & -1.092 \times 10^{13} & -1.092 \times 10^{14} \end{pmatrix}$$


10.

1.00187

ab2 = {ab[[2]], ab[[1]]};
MatrixForm[ab2]
ab1 = {ab2[[1]], ab2[[2]] - ab2[[1]] * (ab2[[2]][[1]] / ab2[[1]][[1]])};
MatrixForm[ab1]
x2sol = ab1[[2]][[3]] / ab1[[2]][[2]]
x1sol = (ab1[[1]][[3]] - ab1[[1]][[2]] * x2sol) / ab1[[1]][[1]]


$$\begin{pmatrix} 5.2 & -3.1 & -25.8 \\ 1. \times 10^{-10} & 2.1 & 21. \end{pmatrix}$$



$$\begin{pmatrix} 5.2 & -3.1 & -25.8 \\ 0. & 2.1 & 21. \end{pmatrix}$$


10.

1.

(* ESERCIZIO 6.2.1 (a) *)

(* Eliminazione normale *)

```

```

Clear[a, ab, ab1, b]
a = {{1, -5, 1}, {10, 0, 20}, {5, 0, -1}};
b = {7, 6, 4};
ab = {Join[a[[1]], {b[[1]]}], Join[a[[2]], {b[[2]]}], Join[a[[3]], {b[[3]]}}};
x = {x1, x2, x3};
MatrixForm[a]
MatrixForm[b]
MatrixForm[ab]


$$\begin{pmatrix} 1 & -5 & 1 \\ 10 & 0 & 20 \\ 5 & 0 & -1 \end{pmatrix}$$



$$\begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix}$$



$$\begin{pmatrix} 1 & -5 & 1 & 7 \\ 10 & 0 & 20 & 6 \\ 5 & 0 & -1 & 4 \end{pmatrix}$$


sys = Dot[a, x] == b
Solve[sys, x]
% // N

{x1 - 5 x2 + x3, 10 x1 + 20 x3, 5 x1 - x3} == {7, 6, 4}


$$\left\{ \left\{ x1 \rightarrow \frac{43}{55}, x2 \rightarrow -\frac{347}{275}, x3 \rightarrow -\frac{1}{11} \right\} \right\}$$


{x1 → 0.781818, x2 → -1.26182, x3 → -0.0909091}

ab1 = {ab[[1]], ab[[2]] - ab[[2]][[1]] / ab[[1]][[1]] * ab[[1]],
       ab[[3]] - ab[[3]][[1]] / ab[[1]][[1]] * ab[[1]]};
MatrixForm[
  ab1]


$$\begin{pmatrix} 1 & -5 & 1 & 7 \\ 0 & 50 & 10 & -64 \\ 0 & 25 & -6 & -31 \end{pmatrix}$$


ab2 = {ab1[[1]], ab1[[2]], ab1[[3]] - ab1[[3]][[2]] / ab1[[2]][[2]] * ab1[[2]]};
MatrixForm[ab2]


$$\begin{pmatrix} 1 & -5 & 1 & 7 \\ 0 & 50 & 10 & -64 \\ 0 & 0 & -11 & 1 \end{pmatrix}$$


x3sol = ab2[[3]][[4]] / ab2[[3]][[3]]
x2sol = (ab2[[2]][[4]] - ab2[[2]][[3]] * x3sol) / ab2[[2]][[2]]
x1sol = (ab2[[1]][[4]] - ab2[[1]][[3]] * x3sol - ab2[[1]][[2]] * x2sol) / ab2[[1]][[1]]


$$-\frac{1}{11}$$



$$-\frac{347}{275}$$



$$\frac{43}{55}$$


(* Con pivoting parziale *)

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```

Clear[a, ab, ab1, b]
a = {{1, -5, 1}, {10, 0, 20}, {5, 0, -1}};
b = {7, 6, 4};
ab = {Join[a[[1]], {b[[1]]}], Join[a[[2]], {b[[2]]}], Join[a[[3]], {b[[3]]}}};
x = {x1, x2, x3};
MatrixForm[a]
MatrixForm[b]
MatrixForm[ab]


$$\begin{pmatrix} 1 & -5 & 1 \\ 10 & 0 & 20 \\ 5 & 0 & -1 \end{pmatrix}$$



$$\begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix}$$



$$\begin{pmatrix} 1 & -5 & 1 & 7 \\ 10 & 0 & 20 & 6 \\ 5 & 0 & -1 & 4 \end{pmatrix}$$


ab1 = {ab[[2]], ab[[1]], ab[[3]]};
MatrixForm[ab1]


$$\begin{pmatrix} 10 & 0 & 20 & 6 \\ 1 & -5 & 1 & 7 \\ 5 & 0 & -1 & 4 \end{pmatrix}$$


ab2 = {ab1[[1]], ab1[[2]] - ab1[[2]][[1]] / ab1[[1]][[1]] * ab1[[1]],
      ab1[[3]] - ab1[[3]][[1]] / ab1[[1]][[1]] * ab1[[1]]};
MatrixForm[
  ab2]


$$\begin{pmatrix} 10 & 0 & 20 & 6 \\ 0 & -5 & -1 & \frac{32}{5} \\ 0 & 0 & -11 & 1 \end{pmatrix}$$


x3sol = ab2[[3]][[4]] / ab2[[3]][[3]]
x2sol = (ab2[[2]][[4]] - ab2[[2]][[3]] * x3sol) / ab2[[2]][[2]]
x1sol = (ab2[[1]][[4]] - ab2[[1]][[3]] * x3sol - ab2[[1]][[2]] * x2sol) / ab2[[1]][[1]]


$$-\frac{1}{11}$$



$$-\frac{347}{275}$$



$$\frac{43}{55}$$


(* Con pivoting parziale scalato *)

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```

Clear[a, ab, ab1, b]
a = {{1, -5, 1}, {10, 0, 20}, {5, 0, -1}};
b = {7, 6, 4};
ab = {Join[a[[1]], {b[[1]]}], Join[a[[2]], {b[[2]]}], Join[a[[3]], {b[[3]]}}};
x = {x1, x2, x3};
MatrixForm[a]
MatrixForm[b]
MatrixForm[ab]

```

$$\begin{pmatrix} 1 & -5 & 1 \\ 10 & 0 & 20 \\ 5 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -5 & 1 & 7 \\ 10 & 0 & 20 & 6 \\ 5 & 0 & -1 & 4 \end{pmatrix}$$

```

Do[s[i] = Max[Table[Abs[a[[i]][[j]]], {j, 1, 3}], {i, 1, 3}]
Do[r[i] = Abs[a[[i]][[1]]] / s[i], {i, 1, 3}]

```

```

s[1]
s[2]
s[3]

```

5

20

5

```

r[1]
r[2]
r[3]

```

$$\frac{1}{5}$$

$$\frac{1}{20}$$

1

```

ab1 = {ab[[3]], ab[[2]], ab[[1]]};
MatrixForm[ab1]

```

$$\begin{pmatrix} 5 & 0 & -1 & 4 \\ 10 & 0 & 20 & 6 \\ 1 & -5 & 1 & 7 \end{pmatrix}$$

```

ab2 = {ab1[[1]], ab1[[2]] - ab1[[2]][[1]] / ab1[[1]][[1]] * ab1[[1]],
       ab1[[3]] - ab1[[3]][[1]] / ab1[[1]][[1]] * ab1[[1]]};

```

```

MatrixForm[
ab2]

```

$$\begin{pmatrix} 5 & 0 & -1 & 4 \\ 0 & 0 & 22 & -2 \\ 0 & -5 & \frac{6}{5} & \frac{31}{5} \end{pmatrix}$$

```
ab3 = {ab2[[1]], ab2[[3]], ab2[[2]]};
MatrixForm[ab3]
```

$$\begin{pmatrix} 5 & 0 & -1 & 4 \\ 0 & -5 & \frac{6}{5} & \frac{31}{5} \\ 0 & 0 & 22 & -2 \end{pmatrix}$$

```
x3sol = ab3[[3]][[4]] / ab3[[3]][[3]]
```

```
x2sol = (ab3[[2]][[4]] - ab3[[2]][[3]] * x3sol) / ab3[[2]][[2]]
```

```
x1sol = (ab3[[1]][[4]] - ab3[[1]][[3]] * x3sol - ab3[[1]][[2]] * x2sol) / ab3[[1]][[1]]
```

$$-\frac{1}{11}$$

$$-\frac{347}{275}$$

$$\frac{43}{55}$$