

```
Quit[];
```

```
(* FATTORIZZAZIONE LU DI UNA MATRICE *)
```

```
Clear[a]; n = 4;
```

```
a = {{1, -3, 4, 5}, {2, 3, 4, -1}, {5, -5, 2, 3}, {-1, -2, 7, 6}};
```

```
MatrixForm[a]
```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 2 & 3 & 4 & -1 \\ 5 & -5 & 2 & 3 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

```
a1 = a;
```

```
Do[m[j][1] = a1[[j]][[1]] / a1[[1]][[1]]; Print[m[j][1]], {j, 2, n}]
```

2

5

-1

```
a2 = {a1[[1]], a1[[2]] - m[2][1] * a1[[1]],
```

```
      a1[[3]] - m[3][1] * a1[[1]], a1[[4]] - m[4][1] * a1[[1]]};
```

```
MatrixForm[
```

```
  a2]
```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 9 & -4 & -11 \\ 0 & 10 & -18 & -22 \\ 0 & -5 & 11 & 11 \end{pmatrix}$$

```
Do[m[j][2] = a2[[j]][[2]] / a2[[2]][[2]]; Print[m[j][2]], {j, 3, n}];
```

10

9

5

-9

```
a3 = {a2[[1]], a2[[2]], a2[[3]] - m[3][2] * a2[[2]], a2[[4]] - m[4][2] * a2[[2]]};
```

```
MatrixForm[a3]
```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 9 & -4 & -11 \\ 0 & 0 & -\frac{122}{9} & -\frac{88}{9} \\ 0 & 0 & \frac{79}{9} & \frac{44}{9} \end{pmatrix}$$

```
Do[m[j][3] = a3[[j]][[3]] / a3[[3]][[3]]; Print[m[j][3]], {j, 4, n}]
```

79

122

```

a4 = {a3[[1]], a3[[2]], a3[[3]], a3[[4]] - m[4][3] * a3[[3]]};
MatrixForm[a2]
MatrixForm[a3]
MatrixForm[a4]
U = a4;

```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 9 & -4 & -11 \\ 0 & 10 & -18 & -22 \\ 0 & -5 & 11 & 11 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 9 & -4 & -11 \\ 0 & 0 & -\frac{122}{9} & -\frac{88}{9} \\ 0 & 0 & \frac{79}{9} & \frac{44}{9} \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 9 & -4 & -11 \\ 0 & 0 & -\frac{122}{9} & -\frac{88}{9} \\ 0 & 0 & 0 & -\frac{88}{61} \end{pmatrix}$$

```

Do[l1[i][j] = 0, {i, 1, n}, {j, 1, n}];
Do[l1[i][i] = 1, {i, 1, n}];
Do[l1[i][j] = m[i][j], {i, 2, n}, {j, 1, i - 1}];

```

```

L = Table[l1[i][j], {i, 1, n}, {j, 1, n}];
MatrixForm[L]

```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 5 & \frac{10}{9} & 1 & 0 \\ -1 & -\frac{5}{9} & -\frac{79}{122} & 1 \end{pmatrix}$$

```

LU = Dot[L, U];
MatrixForm[LU]
MatrixForm[a]

```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 2 & 3 & 4 & -1 \\ 5 & -5 & 2 & 3 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 2 & 3 & 4 & -1 \\ 5 & -5 & 2 & 3 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

(\* FATTORIZZAZIONE CON SCAMBIO \*)

```

Clear[a, a1, a2, a3, a4]
n = 4;
a = {{1, -3, 4, 5}, {2, -6, 4, -1}, {5, -5, 2, 3}, {-1, -2, 7, 6}};
MatrixForm[a]

```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 2 & -6 & 4 & -1 \\ 5 & -5 & 2 & 3 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

```

a1 = a;
Do[m[j][1] = a1[[j]][[1]] / a1[[1]][[1]], {j, 2, n}];
a2 = {a1[[1]], a1[[2]] - m[2][1] * a1[[1]],
      a1[[3]] - m[3][1] * a1[[1]], a1[[4]] - m[4][1] * a1[[1]]};
MatrixForm[
a2]

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 0 & -4 & -11 \\ 0 & 10 & -18 & -22 \\ 0 & -5 & 11 & 11 \end{pmatrix}$$

P = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}};
MatrixForm[P]

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a1 = Dot[P, a];
MatrixForm[a1]

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 5 & -5 & 2 & 3 \\ 2 & -6 & 4 & -1 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

Do[m[j][1] = a1[[j]][[1]] / a1[[1]][[1]], {j, 2, n}]
a2 = {a1[[1]], a1[[2]] - m[2][1] * a1[[1]],
      a1[[3]] - m[3][1] * a1[[1]], a1[[4]] - m[4][1] * a1[[1]]};
Do[m[j][2] = a2[[j]][[2]] / a2[[2]][[2]], {j, 3, n}];
a3 = {a2[[1]], a2[[2]], a2[[3]] - m[3][2] * a2[[2]], a2[[4]] - m[4][2] * a2[[2]]};
Do[m[j][3] = a3[[j]][[3]] / a3[[3]][[3]], {j, 4, n}];
a4 = {a3[[1]], a3[[2]], a3[[3]], a3[[4]] - m[4][3] * a3[[3]]};
MatrixForm[a2]
MatrixForm[a3]
MatrixForm[a4]
U = a4;

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 10 & -18 & -22 \\ 0 & 0 & -4 & -11 \\ 0 & -5 & 11 & 11 \end{pmatrix}$$


$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 10 & -18 & -22 \\ 0 & 0 & -4 & -11 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$


$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 0 & 10 & -18 & -22 \\ 0 & 0 & -4 & -11 \\ 0 & 0 & 0 & -\frac{11}{2} \end{pmatrix}$$

Do[ll[i][j] = 0, {i, 1, n}, {j, 1, n}];
Do[ll[i][i] = 1, {i, 1, n}];
Do[ll[i][j] = m[i][j], {i, 2, n}, {j, 1, i - 1}];

```

```
L = Table[l1[i][j], {i, 1, n}, {j, 1, n}];
MatrixForm[L]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -1 & -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix}$$

```
LU = Dot[L, U];
MatrixForm[LU]
```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 5 & -5 & 2 & 3 \\ 2 & -6 & 4 & -1 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

```
MatrixForm[Dot[P, a]]
```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 5 & -5 & 2 & 3 \\ 2 & -6 & 4 & -1 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

```
MatrixForm[a]
```

$$\begin{pmatrix} 1 & -3 & 4 & 5 \\ 2 & -6 & 4 & -1 \\ 5 & -5 & 2 & 3 \\ -1 & -2 & 7 & 6 \end{pmatrix}$$

(\* ESERCIZIO 6.5.2 \*)

```
Clear[a, a1, a2, a3, a4]
n = 3;
a = {{1, 2, -1}, {2, 4, 0}, {0, 1, -1}};
a = a;
MatrixForm[a]
```

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

```
P = {{1, 0, 0}, {0, 0, 1}, {0, 1, 0}};
MatrixForm[P]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

```
a1 = Dot[P, a];
MatrixForm[a1]
```

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 2 & 4 & 0 \end{pmatrix}$$

```
m[3][1] = a1[[3]][[1]] / a1[[1]][[1]]
```

```
a2 = {a1[[1]], a1[[2]], a1[[3]] - m[3][1] * a1[[1]]};
MatrixForm[a2]
U = a2;
```

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

```
Do[l1[i][j] = 0, {i, 1, n}, {j, 1, n}];
Do[l1[i][i] = 1, {i, 1, n}];
Do[l1[i][j] = m[i][j], {i, 3, n}, {j, 1, i - 1}];
```

```
L = Table[l1[i][j], {i, 1, n}, {j, 1, n}];
MatrixForm[L]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

```
LU = Dot[L, U];
MatrixForm[LU]
MatrixForm[a1]
```

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 2 & 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 2 & 4 & 0 \end{pmatrix}$$

(\* ESERCIZIO 6.5.4 \*)

```
Clear[a, a1, a2, a3, a4];
n = 3;
a = {{2, -1, 1}, {3, 3, 9}, {3, 3, 5}};
MatrixForm[a]
```

$$\begin{pmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{pmatrix}$$

```
a1 = a;
Do[m[j][1] = a1[[j]][[1]] / a1[[1]][[1]], {j, 2, n}];
a2 = {a1[[1]], a1[[2]] - m[2][1] * a1[[1]], a1[[3]] - m[3][1] * a1[[1]]};
Do[m[j][2] = a2[[j]][[2]] / a2[[2]][[2]], {j, 3, n}];
a3 = {a2[[1]], a2[[2]], a2[[3]] - m[3][2] * a2[[2]]};
MatrixForm[a2]
MatrixForm[a3]
U = a3;
```

$$\begin{pmatrix} 2 & -1 & 1 \\ 0 & \frac{9}{2} & \frac{15}{2} \\ 0 & \frac{9}{2} & \frac{7}{2} \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 \\ 0 & \frac{9}{2} & \frac{15}{2} \\ 0 & 0 & -4 \end{pmatrix}$$

```
Do[l1[i][j] = 0, {i, 1, n}, {j, 1, n}];
Do[l1[i][i] = 1, {i, 1, n}];
Do[l1[i][j] = m[i][j], {i, 2, n}, {j, 1, i - 1}];
```

```
L = Table[l1[i][j], {i, 1, n}, {j, 1, n}];
MatrixForm[L]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{3}{2} & 1 & 1 \end{pmatrix}$$

```
LU = Dot[L, U];
MatrixForm[LU]
MatrixForm[a]
```

$$\begin{pmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{pmatrix}$$

```
b = {-1, 0, 4};
x = {x1, x2, x3};
y = {y1, y2, y3};
sys = Dot[L, y] == b
Thread[sys]
```

$$\left\{ y_1, \frac{3 y_1}{2} + y_2, \frac{3 y_1}{2} + y_2 + y_3 \right\} == \{-1, 0, 4\}$$

$$\left\{ y_1 == -1, \frac{3 y_1}{2} + y_2 == 0, \frac{3 y_1}{2} + y_2 + y_3 == 4 \right\}$$

```
y1sol = b[[1]] / L[[1]][[1]]
y2sol = (b[[2]] - L[[2]][[1]] * y1sol) / L[[2]][[2]]
y3sol = (b[[3]] - L[[3]][[1]] * y1sol - L[[3]][[2]] * y2sol) / L[[3]][[3]]
```

```
-1
```

$$\frac{3}{2}$$

```
4
```

```
sol = {y1 → y1sol, y2 → y2sol, y3 → y3sol}
```

$$\left\{ y_1 \rightarrow -1, y_2 \rightarrow \frac{3}{2}, y_3 \rightarrow 4 \right\}$$

```
sys1 = Dot[U, x] == y /. sol
```

$$\left\{ 2 x_1 - x_2 + x_3, \frac{9 x_2}{2} + \frac{15 x_3}{2}, -4 x_3 \right\} == \left\{ -1, \frac{3}{2}, 4 \right\}$$

```
x3sol = y3sol / U[[n]][[n]]
x2sol = (y2sol - U[[2]][[3]] * x3sol) / U[[n-1]][[n-1]]
x1sol = (y1sol - U[[1]][[3]] * x3sol - U[[1]][[2]] * x2sol) / U[[n-2]][[n-2]]
```

```
-1
```

```
2
```

```
1
```

```
Solve[Dot[a, x] == b, x]
```

$$\{ \{ x_1 \rightarrow 1, x_2 \rightarrow 2, x_3 \rightarrow -1 \} \}$$