```
Quit[];

(* AUTOVALORI ED AUTOVETTORI *)

(* Esempio 1. *)

a = {{1, 2, -1}, {4, 1, -1}, {2, 0, 1}};
id = IdentityMatrix[3];
MatrixForm[a]
Eigenvalues[a]
Eigenvectors[a]
```

$$\begin{pmatrix} 1 & 2 & -1 \\ 4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix}$$

$$\left\{ 3, -\sqrt{3}, \sqrt{3} \right\}$$

$$\left\{ \{2, 3, 2\}, \left\{ -\frac{1}{2} - \frac{\sqrt{3}}{2}, \frac{3}{2} + \frac{\sqrt{3}}{2}, 1 \right\}, \left\{ -\frac{1}{2} + \frac{\sqrt{3}}{2}, \frac{3}{2} - \frac{\sqrt{3}}{2}, 1 \right\} \right\}$$

```
a1 = a - λ * id;
MatrixForm[a1]
eq = Det[a1] == 0
eval = Solve[eq, λ]
```

$$\begin{pmatrix} 1-\lambda & 2 & -1 \\ 4 & 1-\lambda & -1 \\ 2 & 0 & 1-\lambda \end{pmatrix}$$

$$-9 + 3\lambda + 3\lambda^2 - \lambda^3 == 0$$

$$\left\{ \{\lambda \to 3\}, \left\{ \lambda \to -\sqrt{3} \right\}, \left\{ \lambda \to \sqrt{3} \right\} \right\}$$

```
eval[[1]]
eval[[2]]
eval[[3]]
```

$$\{\lambda \to 3\}$$

$$\left\{ \lambda \to -\sqrt{3} \right\}$$

$$\left\{ \lambda \to \sqrt{3} \right\}$$

```
x = {x1, x2, x3};
sys1 = Dot[(a1 /. eval[[1]]), x] == 0
sol1 = Solve[sys1, x][[1]]
sol1 /. x3 → 2
```

$$\{-2 x1 + 2 x2 - x3, 4 x1 - 2 x2 - x3, 2 x1 - 2 x3\} == 0$$

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{ x1 \to x3, x2 \to \frac{3 x3}{2} \right\}$$

$$\{x1 \to 2, x2 \to 3\}$$

```
sys2 = Dot[(a1 /. eval[[2]]), x] == 0;
sol2 = Solve[sys2, x][[1]]
sol2 /. x3 → 1
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{x2 \to -\frac{1}{2}\left(-3-\sqrt{3}\right)x3,\ x1 \to -\frac{1}{2}\left(1+\sqrt{3}\right)x3\right\}$$

$$\left\{x2 \to \frac{1}{2}\left(3+\sqrt{3}\right),\ x1 \to \frac{1}{2}\left(-1-\sqrt{3}\right)\right\}$$

```
sys3 = Dot[(a1 /. eval[[3]]), x] == 0;
sol3 = Solve[sys3, x][[1]]
sol3 /. x3 → 1
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{x2 \to -\frac{1}{2}\left(-3+\sqrt{3}\right)x3,\ x1 \to -\frac{1}{2}\left(1-\sqrt{3}\right)x3\right\}$$

$$\left\{x2 \to \frac{1}{2}\left(3-\sqrt{3}\right),\ x1 \to \frac{1}{2}\left(-1+\sqrt{3}\right)\right\}$$

```
(* Secondo esempio *)

Clear[a];
a = {{0, 1}, {-1, 0}};
id = IdentityMatrix[3];
MatrixForm[a]
Eigenvalues[a]
```

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$\{i, -i\}$

```
(* Terzo esempio *)

Clear[a];
a = {{1, 1}, {-1, 2}};
id = IdentityMatrix[3];
MatrixForm[a]
Eigenvalues[a]
```

$$\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$\left\{\frac{1}{2}\left(3+i\sqrt{3}\right),\ \frac{1}{2}\left(3-i\sqrt{3}\right)\right\}$$

```
(* Quarto esempio *)

Clear[a];
a = {{1, 1, 0}, {-1, 2, 0}, {0, 0, 3}};
id = IdentityMatrix[3];
MatrixForm[a]
Eigenvalues[a]
```

$$\begin{pmatrix} 1 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\left\{3,\ \frac{1}{2}\left(3+i\sqrt{3}\right),\ \frac{1}{2}\left(3-i\sqrt{3}\right)\right\}$$

```
(* Quinto esempio *)
```

```
Clear[a];
a = {{1, 2, 3}, {-1, -2, -3}, {4, 5, 1}};
id = IdentityMatrix[3];
MatrixForm[a]
Eigenvalues[a]
```

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 4 & 5 & 1 \end{pmatrix}$$

$$\left\{ i \sqrt{2}, -i \sqrt{2}, 0 \right\}$$