

Quit[]

(** Esercizio 4.3.9 **)

n = 3;

f[x_] := Sum[a[k] * x^k, {k, 0, n}]

f[x]

a[0] + x a[1] + x^2 a[2] + x^3 a[3]

integrale = Integrate[f[x], {x, -1, 1}]

quadratura = f[-√3 / 3] + f[√3 / 3]

integrale - quadratura

$2 a[0] + \frac{2 a[2]}{3}$

$2 a[0] + \frac{2 a[2]}{3}$

0

(* Era necessario ? *)

f[x_] := x^4

f[x]

integrale = Integrate[f[x], {x, -1, 1}]

quadratura = f[-√3 / 3] + f[√3 / 3]

integrale - quadratura

x^4

$\frac{2}{5}$

$\frac{2}{9}$

$\frac{8}{45}$

(** Esercizio 4.3.10 **)

n = 3;

f[x_] := Sum[a[k] * x^k, {k, 0, n}]

f[x]

a[0] + x a[1] + x^2 a[2] + x^3 a[3]

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h = (b - a) / 3;
x[0] = a;
x[1] = a + h;
x[2] = b;
integrale = Integrate[f[x], {x, a, b}]
quadratura = 3 * h / 4 * (3 * f[x[1]] + f[x[2]])
integrale - quadratura

-a a[0] + b a[0] -  $\frac{1}{2} a^2 a[1] + \frac{1}{2} b^2 a[1] - \frac{1}{3} a^3 a[2] + \frac{1}{3} b^3 a[2] - \frac{1}{4} a^4 a[3] + \frac{1}{4} b^4 a[3]$ 

 $\frac{1}{4} (-a + b) \left( a[0] + b a[1] + b^2 a[2] + b^3 a[3] + \right.$ 
 $3 \left( a[0] + \left( a + \frac{1}{3} (-a + b) \right) a[1] + \left( a + \frac{1}{3} (-a + b) \right)^2 a[2] + \left( a + \frac{1}{3} (-a + b) \right)^3 a[3] \right) \left. \right)$ 

-a a[0] + b a[0] -  $\frac{1}{2} a^2 a[1] + \frac{1}{2} b^2 a[1] - \frac{1}{3} a^3 a[2] + \frac{1}{3} b^3 a[2] -$ 
 $\frac{1}{4} a^4 a[3] + \frac{1}{4} b^4 a[3] - \frac{1}{4} (-a + b) \left( a[0] + b a[1] + b^2 a[2] + b^3 a[3] + \right.$ 
 $3 \left( a[0] + \left( a + \frac{1}{3} (-a + b) \right) a[1] + \left( a + \frac{1}{3} (-a + b) \right)^2 a[2] + \left( a + \frac{1}{3} (-a + b) \right)^3 a[3] \right) \left. \right)$ 

Simplify[integrale - quadratura]

- $\frac{1}{36} (a - b)^4 a[3]$ 

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(Esercizio 4.3.11 **)**

```

Clear[f0, f1, f2, b0, b1, b2];
f0[x_] := 1;
b0 = Integrate[f0[x], {x, -1, 1}]
f1[x_] := x;
b1 = Integrate[f1[x], {x, -1, 1}]
f2[x_] := x^2;
b2 = Integrate[f2[x], {x, -1, 1}]

2
0
 $\frac{2}{3}$ 

Clear[eq0, eq1, eq2, c0, c1, c2];
eq0 = c0 * f0[-1] + c1 * f0[0] + c2 * f0[1] == b0
eq1 = c0 * f1[-1] + c1 * f1[0] + c2 * f1[1] == b1
eq2 = c0 * f2[-1] + c1 * f2[0] + c2 * f2[1] == b2

c0 + c1 + c2 == 2
-c0 + c2 == 0

c0 + c2 ==  $\frac{2}{3}$ 

Solve[{eq0, eq1, eq2}, {c0, c1, c2}]

```

$$\left\{ \left\{ c0 \rightarrow \frac{1}{3}, c1 \rightarrow \frac{4}{3}, c2 \rightarrow \frac{1}{3} \right\} \right\}$$

(Esercizio 4.3.12 **)**

```

Clear[f0, f1, f2, b0, b1, b2];
f0[x_] := 1;
b0 = Integrate[f0[x], {x, 0, 2}]
f1[x_] := x;
b1 = Integrate[f1[x], {x, 0, 2}]
f2[x_] := x^2;
b2 = Integrate[f2[x], {x, 0, 2}]

2

2

8
3

Clear[eq0, eq1, eq2, c0, c1, c2];
eq0 = c0 * f0[0] + c1 * f0[1] + c2 * f0[2] == b0
eq1 = c0 * f1[0] + c1 * f1[1] + c2 * f1[2] == b1
eq2 = c0 * f2[0] + c1 * f2[1] + c2 * f2[2] == b2

c0 + c1 + c2 == 2

c1 + 2 c2 == 2

c1 + 4 c2 == 8
3

Solve[{eq0, eq1, eq2}, {c0, c1, c2}]

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{{c0 -> 1/3, c1 -> 4/3, c2 -> 1/3}}

```

(Esercizio 4.3.13 **)**

```

f0[x_] := 1;
b0 = Integrate[f0[x], {x, 0, 1}]
f1[x_] := x
b1 = Integrate[f1[x], {x, 0, 1}]
f2[x_] := x^2
b2 = Integrate[f2[x], {x, 0, 1}]
f3[x_] := x^3
b3 = Integrate[f3[x], {x, 0, 1}]

1

1
2

1
3

1
4

eq0 = c0 * f0[0] + c1 * f0[x1] == b0
eq1 = c0 * f1[0] + c1 * f1[x1] == b1
eq2 = c0 * f2[0] + c1 * f2[x1] == b2

c0 + c1 == 1

c1 x1 == 1
2

c1 x1^2 == 1
3

```

```
sol = Solve[{eq0, eq1, eq2}, {c0, c1, x1}][[1]]
```

$$\left\{ c0 \rightarrow \frac{1}{4}, c1 \rightarrow \frac{3}{4}, x1 \rightarrow \frac{2}{3} \right\}$$

```
(c0 * f3[0] + c1 * f3[x1]) /. sol
```

$$\frac{2}{9}$$