```
(** ROMBERG  **)

f[x_] := Exp[x];
a = 0.; b = 1;
true1 = Integrate[f[x], x]
true = Integrate[f[x], {x, a, b}]
```

e^x

1.71828

```
nmax = 5;
h[1] = b - a;
r[1][1] = 0.5 * h[1] * (f[a] + f[b]);
Do[n = 2^(k - 1); h[k] = 0.5 * h[k - 1];
 sum = 0.0; Do[sum += f[a + j * h[k]], {j, 1, n, 2}];
 r[k][1] = 0.5 * r[k - 1][1] + h[k] * sum;
 Do[r[k][j] = r[k][j - 1] + (r[k][j - 1] - r[k - 1][j - 1]) / (4^(j - 1) - 1), {j, 2, k}],
 {k, 2, nmax}]
Do[err = Abs[r[n][n] - true]; Print["n = ", n, "    err = ", err], {n, 1, nmax}]
Do[Print["n = ", n, "  j = ", j, "  r = ", r[n][j]], {n, 1, nmax}, {j, 1, n}]
```

n = 1 err = 0.140859

n = 2 err = 0.000579323

n = 3 err = 8.59466×10^{-7}

n = 4 err = 3.35485×10^{-10}

n = 5 err = 3.33067×10^{-14}

n = 1 j = 1 r = 1.85914

n = 2 j = 1 r = 1.75393

n = 2 j = 2 r = 1.71886

n = 3 j = 1 r = 1.72722

n = 3 j = 2 r = 1.71832

n = 3 j = 3 r = 1.71828

n = 4 j = 1 r = 1.72052

n = 4 j = 2 r = 1.71828

n = 4 j = 3 r = 1.71828

n = 4 j = 4 r = 1.71828

n = 5 j = 1 r = 1.71884

n = 5 j = 2 r = 1.71828

n = 5 j = 3 r = 1.71828

n = 5 j = 4 r = 1.71828

n = 5 j = 5 r = 1.71828

```
nmax = 5; tol = 1.0 * 10^(-6);
h[1] = b - a;
r[1][1] = 0.5 * h[1] * (f[a] + f[b]);

k = 2; test = 1.0;
While[test > tol, n = 2^(k - 1); h[k] = 0.5 * h[k - 1];
 sum = 0.0; Do[sum += f[a + j * h[k]], {j, 1, n, 2}];
 r[k][1] = 0.5 * r[k - 1][1] + h[k] * sum;
 Do[r[k][j] = r[k][j - 1] + (r[k][j - 1] - r[k - 1][j - 1]) / (4^(j - 1) - 1), {j, 2, k}];
 test = Abs[r[k][k] - r[k - 1][k - 1]]; test1 = Abs[r[k][k] - true];
 Print["k = ", k, "   test = ", test, "   test1 = ", test1]; k++]
```

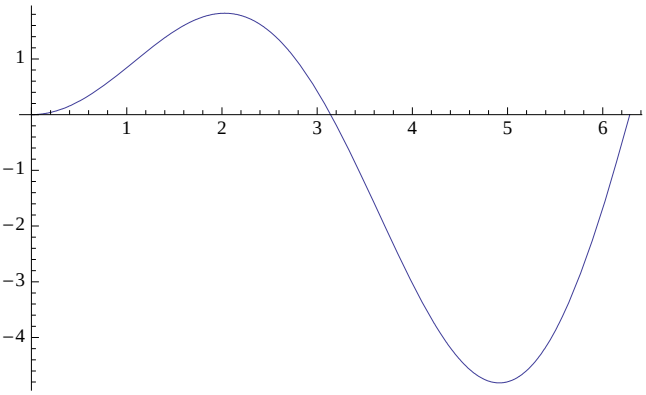
```
k = 2   test = 0.14028   test1 = 0.000579323

k = 3   test = 0.000578464   test1 = 8.59466×10^-7

k = 4   test = 8.5913×10^-7   test1 = 3.35485×10^-10
```

```
(* SECONDA FUNZIONE *)

f[x_] := x * Sin[x];
a = 0.; b = 2. * Pi;
Plot[f[x], {x, 0, b}]
true1 = Integrate[f[x], x]
true = Integrate[f[x], {x, a, b}] // N
```




```
-x Cos[x] + Sin[x]

-6.28319
```

```
nmax = 10;
h[1] = b - a;
r[1][1] = 0.5 * h[1] * (f[a] + f[b]);
Do[n = 2^(k - 1); h[k] = 0.5 * h[k - 1];
 sum = 0.0; Do[sum += f[a + j * h[k]], {j, 1, n, 2}];
 r[k][1] = 0.5 * r[k - 1][1] + h[k] * sum;
 Do[r[k][j] = r[k][j - 1] + (r[k][j - 1] - r[k - 1][j - 1]) / (4^(j - 1) - 1), {j, 2, k}],
 {k, 2, nmax}]
Do[err = Abs[r[n][n] - true]; Print["n = ", n, "    err = ", err], {n, 1, nmax}]
```

```
n = 1    err = 6.28319

n = 2    err = 6.28319

n = 3    err = 0.7352

n = 4    err = 0.0162313

n = 5    err = 0.0000811563

n = 6    err = 9.63528×10^-8

n = 7    err = 2.76792×10^-11

n = 8    err = 1.77636×10^-15

n = 9    err = 1.77636×10^-15

n = 10    err = 8.88178×10^-16
```

```
(* Do[Print["n = ",n,"  j = ", j,"  r = ", r[n][j]],{n,1,nmax},{j,1,n}];*)

nmax = 5; tol = 1.0 * 10^(-6);
h[1] = b - a;
r[1][1] = 0.5 * h[1] * (f[a] + f[b]);

k = 2; test = 1.0;
While[test > tol, n = 2^(k - 1); h[k] = 0.5 * h[k - 1];
 sum = 0.0; Do[sum += f[a + j * h[k]], {j, 1, n, 2}];
 r[k][1] = 0.5 * r[k - 1][1] + h[k] * sum;
 Do[r[k][j] = r[k][j - 1] + (r[k][j - 1] - r[k - 1][j - 1]) / (4^(j - 1) - 1), {j, 2, k}];
 test = Abs[r[k][k] - r[k - 1][k - 1]]; test1 = Abs[r[k][k] - true];
 Print["k = ", k, "   test = ", test, "   test1 = ", test1]; k++]
```

```
k = 2   test = 4.83471×10^-15   test1 = 6.28319
```

```
Do[n = 2^(k - 1); h[k] = 0.5 * h[k - 1];
 sum = 0.0; Do[sum += f[a + j * h[k]], {j, 1, n, 2}];
 r[k][1] = 0.5 * r[k - 1][1] + h[k] * sum;
 Do[r[k][j] = r[k][j - 1] + (r[k][j - 1] - r[k - 1][j - 1]) / (4^(j - 1) - 1), {j, 2, k}],
 {k, 2, nmax}]
Do[err = Abs[r[n][n] - true]; Print["n = ", n, "    err = ", err], {n, 1, nmax}]
```

```
n = 1    err = 6.28319

n = 2    err = 6.28319

n = 3    err = 0.7352

n = 4    err = 0.0162313

n = 5    err = 0.0000811563
```

```
Do[Print["n = ", n, "  j = ", j, "  r = ", r[n][j]], {n, 1, nmax}, {j, 1, n}]
```

n = 1 j = 1 r = -4.83471×10^{-15}

n = 2 j = 1 r = -1.20868×10^{-15}

n = 2 j = 2 r = 0.

n = 3 j = 1 r = −4.9348

n = 3 j = 2 r = −6.57974

n = 3 j = 3 r = −7.01839

n = 4 j = 1 r = −5.95683

n = 4 j = 2 r = −6.29751

n = 4 j = 3 r = −6.2787

n = 4 j = 4 r = −6.26695

n = 5 j = 1 r = −6.20223

n = 5 j = 2 r = −6.28403

n = 5 j = 3 r = −6.28313

n = 5 j = 4 r = −6.2832

n = 5 j = 5 r = −6.28327