```
(* Esercizio 7.3.5 *)

(*a*)
n = 3;
a = {{3, -1, 1}, {3, 6, 2}, {3, 3, 7}};
MatrixForm[a]
```

$$\begin{pmatrix} 3 & -1 & 1 \\ 3 & 6 & 2 \\ 3 & 3 & 7 \end{pmatrix}$$

```
a1 = a[[1 ;; 1, 1 ;; 1]];
a2 = a[[1 ;; 2, 1 ;; 2]];
a3 = a[[1 ;; 3, 1 ;; 3]];
MatrixForm[a1]
MatrixForm[a2]
MatrixForm[a3]
Det[a1]
Det[a2]
Det[a3]
```

$$\begin{pmatrix} 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 \\ 3 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 & 1 \\ 3 & 6 & 2 \\ 3 & 3 & 7 \end{pmatrix}$$

```
3

21

114

b = {1, 0, 4};
x = Table[xx[i], {i, 1, n}];
MatrixForm[a];
Det[a];
```

```
(* Matrici D ed R *)
d = DiagonalMatrix[Diagonal[a]];
r = a - d;
t = -Dot[Inverse[d], r];
MatrixForm[a]
MatrixForm[b]
MatrixForm[d]
MatrixForm[r]
MatrixForm[t]
λ = Eigenvalues[t] // N
```

$$\begin{pmatrix} 3 & -1 & 1 \\ 3 & 6 & 2 \\ 3 & 3 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 1 \\ 3 & 0 & 2 \\ 3 & 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{2} & 0 & -\frac{1}{3} \\ -\frac{3}{7} & -\frac{3}{7} & 0 \end{pmatrix}$$

```
{-0.41932, 0.20966 + 0.113243 i, 0.20966 - 0.113243 i}
```

```
ρ = Max[Abs[λ]]
ω0 = 2. / (1. + Sqrt[1. - ρ^2])
```

```
0.41932
```

```
1.04831
```

```
MatrixForm[b]
sys = Thread[Dot[a, x] == b];
xsol = x /. Solve[sys, x][[1]] // N
Clear[eq, f, ff];
Do[eq[i] = Reduce[sys[[i]], xx[i]], {i, 1, n}];
Do[ff[i] = eq[i][[2]], {i, 1, n}];
f = Table[ff[i], {i, 1, n}];
xk[0] = Table[0.0, {i, 1, n}]
kmax = 200; tol = 1.0×10^(-6);
ω = ω0;
k = 0;
While[k < kmax,
 Do[
  sysold = Table[xx[j] == xk[k][[j]], {j, i + 1, n}];
  sysnew = Table[xx[j] == xtemp[j], {j, 1, i - 1}];
  sys1 = Join[sysnew, sysold];
  sol1 = Solve[sys1, x][[1]];
  (*Print[i,sysold,sysnew,sys1,sol1];*)
  xtemp[i] = ω * (f[[i]] /. sol1 // N) + (1. - ω) * xk[k][[i]];
  , {i, 1, n}];
 xk[k + 1] = Table[xtemp[i], {i, 1, n}];
 norm = Max[Abs[xk[k + 1] - xk[k]]]; Print[k, "  ", norm]; If[norm ≤ tol, Break[]];
 k++]
Print[xk[k]]
```

$$\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$$

{0.0350877, -0.236842, 0.657895}

{0., 0., 0.}

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

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Solve::svars : Equations may not give solutions for all "solve" variables. ≫

General::stop : Further output of Solve::svars will be suppressed during this calculation. ≫

0 0.524328

1 0.264101

2 0.0380552

3 0.01068

4 0.00100263

5 0.000477697

6 0.0000531284

7 0.0000250857

8 3.73918×10^{-6}

9 1.55956×10^{-6}

10 2.85117×10^{-7}

{0.0350875, -0.236842, 0.657895}

```
(*c*)
n = 4;
a = {{10, 5, 0, 0}, {5, 10, -4, 0}, {0, -4, 8, -1}, {0, 0, -1, 5}};
MatrixForm[a]
```

$$\begin{pmatrix} 10 & 5 & 0 & 0 \\ 5 & 10 & -4 & 0 \\ 0 & -4 & 8 & -1 \\ 0 & 0 & -1 & 5 \end{pmatrix}$$

```
a1 = a[[1 ;; 1, 1 ;; 1]];
a2 = a[[1 ;; 2, 1 ;; 2]];
a3 = a[[1 ;; 3, 1 ;; 3]];
MatrixForm[a1]
MatrixForm[a2]
MatrixForm[a3]
Det[a1]
Det[a2]
Det[a3]
Det[a]
```

$$\begin{pmatrix} 10 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 5 & 0 \\ 5 & 10 & -4 \\ 0 & -4 & 8 \end{pmatrix}$$

```
10
```

```
75
```

```
440
```

```
2125
```

```
b = {6, 25, -11, -11};
x = Table[xx[i], {i, 1, n}];
MatrixForm[a];
Det[a];
```

```
(* Matrici D ed R *)
d = DiagonalMatrix[Diagonal[a]];
r = a - d;
t = -Dot[Inverse[d], r];
MatrixForm[a]
MatrixForm[b]
MatrixForm[d]
MatrixForm[r]
MatrixForm[t]
λ = Eigenvalues[t] // N
```

$$\begin{pmatrix} 10 & 5 & 0 & 0 \\ 5 & 10 & -4 & 0 \\ 0 & -4 & 8 & -1 \\ 0 & 0 & -1 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ 25 \\ -11 \\ -11 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 5 & 0 & 0 \\ 5 & 0 & -4 & 0 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{2}{5} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{8} \\ 0 & 0 & \frac{1}{5} & 0 \end{pmatrix}$$

```
{-0.679305, 0.679305, -0.116379, 0.116379}
```

```
ρ = Max[Abs[λ]]
ω0 = 2. / (1. + Sqrt[1. - ρ^2])
```

```
0.679305
```

```
1.1535
```

```
MatrixForm[b]
sys = Thread[Dot[a, x] == b];
xsol = x /. Solve[sys, x][[1]] // N
Clear[eq, f, ff];
Do[eq[i] = Reduce[sys[[i]], xx[i]], {i, 1, n}];
Do[ff[i] = eq[i][[2]], {i, 1, n}];
f = Table[ff[i], {i, 1, n}];
xk[0] = Table[0.0, {i, 1, n}]
kmax = 200; tol = 1.0×10^(-6);
ω = ω0 - 0.01;
ω = 0.5;
k = 0;
While[k < kmax,
 Do[
  sysold = Table[xx[j] == xk[k][[j]], {j, i + 1, n}];
  sysnew = Table[xx[j] == xtemp[j], {j, 1, i - 1}];
  sys1 = Join[sysnew, sysold];
  sol1 = Solve[sys1, x][[1]];
  (*Print[i,sysold,sysnew,sys1,sol1];*)
  xtemp[i] = ω * (f[[i]] /. sol1 // N) + (1. - ω) * xk[k][[i]];
  , {i, 1, n}];
 xk[k + 1] = Table[xtemp[i], {i, 1, n}];
 norm = Max[Abs[xk[k + 1] - xk[k]]]; Print[k, "  ", norm]; If[norm ≤ tol, Break[]];
 k++]
Print[xk[k]]
```

$$\begin{pmatrix} 6 \\ 25 \\ -11 \\ -11 \end{pmatrix}$$

{-0.797647, 2.79529, -0.258824, -2.25176}

{0., 0., 0., 0.}

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

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Solve::svars : Equations may not give solutions for all "solve" variables. ≫

General::stop : Further output of Solve::svars will be suppressed during this calculation. ≫

```
0  1.175

1  0.582879

2  0.297973

3  0.188037

4  0.136267

5  0.100889

6  0.0746768

7  0.0579322

8  0.0457447

9  0.03637

10  0.0290399

11  0.0232497

12  0.0186463

13  0.0149713
```

14 0.0120296

15 0.00967056

16 0.00777668

17 0.00625504

18 0.00503186

19 0.00404826

20 0.00325714

21 0.00262073

22 0.00210873

23 0.00169679

24 0.00136534

25 0.00109865

26 0.000884053

27 0.000711376

28 0.000572428

29 0.000460621

30 0.000370653

31 0.000298257

32 0.000240002

33 0.000193125

34 0.000155405

35 0.000125051

36 0.000100627

37 0.0000809724

38 0.0000651571

39 0.0000524307

40 0.0000421901

41 0.0000339496

42 0.0000273187

43 0.0000219828

44 0.0000176892

45 0.0000142342

46 0.000011454

47 9.21684×10^{-6}

48 7.41663×10^{-6}

49 5.96803×10^{-6}

50 4.80237×10^{-6}

51 3.86438×10^{-6}

52 3.1096×10^{-6}

53 2.50224×10^{-6}

54 2.01351×10^{-6}

55 1.62023×10^{-6}

56 1.30377×10^{-6}

57 1.04912×10^{-6}

58 8.44212×10^{-7}

{-0.797644, 2.79529, -0.258827, -2.25177}

```
(* Esercizio 7.3.10 *)

(*a*)
n = 3;
a = {{1, 2, -2}, {1, 1, 1}, {2, 2, 1}};
MatrixForm[a]
```

$$\begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix}$$

```
a1 = a[[1 ;; 1, 1 ;; 1]];
a2 = a[[1 ;; 2, 1 ;; 2]];
a3 = a[[1 ;; 3, 1 ;; 3]];
MatrixForm[a1]
MatrixForm[a2]
MatrixForm[a3]
Det[a1]
Det[a2]
Det[a3]
```

(1)

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix}$$

1

-1

1

```
b = {7, 2, 5};
x = Table[xx[i], {i, 1, n}];
MatrixForm[a];
Det[a]
```

1

```
(* Matrici D ed R *)
d = DiagonalMatrix[Diagonal[a]];
r = a - d;
t = -Dot[Inverse[d], r];
MatrixForm[a]
MatrixForm[b]
MatrixForm[d]
MatrixForm[r]
MatrixForm[t]
λ = Eigenvalues[t] // N
```

$$\begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & -2 \\ 1 & 0 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -2 & 2 \\ -1 & 0 & -1 \\ -2 & -2 & 0 \end{pmatrix}$$

```
{0., 0., 0.}
```

```
ρJ = Max[Abs[λ]]
```

```
0.
```

```
(* Matrici D, L ed U *)
d = DiagonalMatrix[Diagonal[a]];
Clear[l, u];
l = Table[0.0, {i, 1, n}, {j, 1, n}];
u = Table[0.0, {i, 1, n}, {j, 1, n}];
Do[l[[i, j]] = a[[i]][[j]], {i, 1, n}, {j, 1, i - 1}];
Do[u[[i, j]] = a[[i]][[j]], {i, 1, n}, {j, i + 1, n}];
t = Dot[Inverse[d - l], u];
```

```
MatrixForm[a]
MatrixForm[b]
MatrixForm[d]
MatrixForm[l]
MatrixForm[u]
MatrixForm[t]
λG = Eigenvalues[t] // N
```

$$\begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0. & 0. & 0. \\ 1 & 0. & 0. \\ 2 & 2 & 0. \end{pmatrix}$$

$$\begin{pmatrix} 0. & 2 & -2 \\ 0. & 0. & 1 \\ 0. & 0. & 0. \end{pmatrix}$$

$$\begin{pmatrix} 0. & 2. & -2. \\ 0. & 2. & -1. \\ 0. & 8. & -6. \end{pmatrix}$$

```
{-4.82843, 0.828427, 0.}
```

```
ρG = Max[Abs[λG]]
```

```
4.82843
```

```
sys = Thread[Dot[a, x] == b];
xsol = x /. Solve[sys, x][[1]] // N
Clear[eq, f, ff]
Do[eq[i] = Reduce[sys[[i]], xx[i]], {i, 1, n}];
Do[ff[i] = eq[i][[2]], {i, 1, n}];
f = Table[ff[i], {i, 1, n}];
xk[0] = Table[0.0, {i, 1, n}];
kmax = 25; tol = 1.0×10^(-4);
k = 0;
While[k < kmax,
 Do[
  sysold = Table[xx[j] == xk[k][[j]], {j, i + 1, n}];
  sysnew = Table[xx[j] == xtemp[j], {j, 1, i - 1}];
  sys1 = Join[sysnew, sysold];
  sol1 = Solve[sys1, x][[1]];
  (*Print[i,sysold,sysnew,sys1,sol1];*)
  xtemp[i] = f[[i]] /. sol1 // N;
  , {i, 1, n}];
 xk[k + 1] = Table[xtemp[i], {i, 1, n}];
 norm = Max[Abs[xk[k + 1] - xk[k]]]; Print[k, "  ", norm]; If[norm ≤ tol, Break[]];
 k++]
Print[xk[k]]
```

```
{1., 2., -1.}
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

General::stop : Further output of Solve::svars will be suppressed during this calculation. ≫

0 7.

1 13.

2 32.

3 76.

4 176.

5 400.

6 896.

7 1984.

8 4352.

9 9472.

10 20480.

11 44032.

12 94208.

13 200704.

14 425984.

15 901120.

16 1.90054×10^6

17 3.9977×10^6

18 8.38861×10^6

19 1.75636×10^7

20 3.67002×10^7

21 7.6546×10^7

22 1.59384×10^8

23 3.3135×10^8

24 6.87866×10^8

$\{1.30862 \times 10^9, -1.3254 \times 10^9, 3.35544 \times 10^7\}$

```
(* Esercizio 7.3.14 *)

n = 10;
x = Table[xx[i], {i, 1, n - 1}]
```

{xx[1], xx[2], xx[3], xx[4], xx[5], xx[6], xx[7], xx[8], xx[9]}

```
a = Table[0.0, {i, 1, n - 1}, {j, 1, n - 1}];
b = Table[0.0, {i, 1, n - 1}];
Do[a[[i, i]] = 1.0, {i, 1, n - 1}];
Do[a[[i, i - 1]] = -0.5; a[[i, i + 1]] = -0.5, {i, 2, n - 2}];
a[[1, 2]] = a[[n - 1, n - 2]] = -0.5;
b[[1]] = 0.5;

sys = Thread[Dot[a, x] == b];
xsol = x /. Solve[sys, x][[1]] // N
```

{0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1}

```
(* Matrici D ed R *)
d = DiagonalMatrix[Diagonal[a]];
r = a - d;
t = -Dot[Inverse[d], r];
MatrixForm[a]
MatrixForm[b]
MatrixForm[d]
MatrixForm[r]
MatrixForm[t]
λ = Eigenvalues[t] // N
ρ = Max[Abs[λ]]
ω0 = 2. / (1. + Sqrt[1. - ρ^2])
```

$$\begin{pmatrix} 1. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ -0.5 & 1. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.5 & 1. & -0.5 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.5 & 1. & -0.5 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & -0.5 & 1. & -0.5 & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & -0.5 & 1. & -0.5 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & -0.5 & 1. & -0.5 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 1. & -0.5 \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 1. \end{pmatrix}$$

$$\begin{pmatrix} 0.5 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} 0. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ -0.5 & 0. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.5 & 0. & -0.5 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.5 & 0. & -0.5 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & -0.5 & 0. & -0.5 & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & -0.5 & 0. & -0.5 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & -0.5 & 0. & -0.5 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 0. & -0.5 \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 0. \end{pmatrix}$$

$$\begin{pmatrix} 0. & 0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0.5 & 0. & 0.5 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0.5 & 0. & 0.5 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0. & 0.5 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0.5 & 0. & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0.5 & 0. & 0.5 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0.5 & 0. & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0.5 & 0. & 0.5 \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.5 & 0. \end{pmatrix}$$

$$\{-0.951057, 0.951057, -0.809017, 0.809017, -0.587785, 0.587785, -0.309017, 0.309017, -4.13144 \times 10^{-17}\}$$

0.951057

1.52786

```
Clear[eq, f, ff]
Do[eq[i] = Reduce[sys[[i]], xx[i]], {i, 1, n - 1}];
Do[ff[i] = eq[i][[2]], {i, 1, n - 1}];
f = Table[ff[i], {i, 1, n - 1}]
```

$$\{0.5 + 0.5\,xx[2],\ 0. + 0.5\,xx[1] + 0.5\,xx[3],\ 0. + 0.5\,xx[2] + 0.5\,xx[4],\ 0. + 0.5\,xx[3] + 0.5\,xx[5],\ 0. + 0.5\,xx[4] + 0.5\,xx[6],\ 0. + 0.5\,xx[5] + 0.5\,xx[7],\ 0. + 0.5\,xx[6] + 0.5\,xx[8],\ 0. + 0.5\,xx[7] + 0.5\,xx[9],\ 0. + 0.5\,xx[8]\}$$

```
xk[0] = Table[0.0, {i, 1, n - 1}]
kmax = 10; tol = 1.0 × 10^(-4);
k = 0;
While[k < kmax, sys1 = Table[xx[i] == xk[k][[i]], {i, 1, n - 1}];
sol1 = Solve[sys1, x][[1]]; xk[k + 1] = f /. sol1 // N;
 norm = Max[Abs[xk[k + 1] - xk[k]]]; Print[k, "  ", norm]; If[norm ≤ tol, Break[]];
 k++]
Print[xk[k]]
```

$$\{0., 0., 0., 0., 0., 0., 0., 0., 0.\}$$

```
0  0.5

1  0.25

2  0.125

3  0.125

4  0.09375

5  0.078125

6  0.0703125

7  0.0546875

8  0.0546875

9  0.046875
```

$$\{0.753906, 0.548828, 0.34375, 0.226563, 0.109375, 0.0654297, 0.0214844, 0.0117188, 0.00195313\}$$

```
(* Matrici D, L ed U *)
d = DiagonalMatrix[Diagonal[a]];
Clear[l, u];
l = Table[0.0, {i, 1, n - 1}, {j, 1, n - 1}];
u = Table[0.0, {i, 1, n - 1}, {j, 1, n - 1}];
Do[l[[i, j]] = a[[i]][[j]], {i, 1, n - 1}, {j, 1, i - 1}];
Do[u[[i, j]] = a[[i]][[j]], {i, 1, n - 1}, {j, i + 1, n - 1}];
t = Dot[Inverse[d - l], u];
```

```
MatrixForm[a]
MatrixForm[b]
MatrixForm[d]
MatrixForm[l]
MatrixForm[u]
MatrixForm[t]
λ = Eigenvalues[t] // N
ρ = Max[Abs[λ]]
```

$$\begin{pmatrix} 1. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ -0.5 & 1. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.5 & 1. & -0.5 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.5 & 1. & -0.5 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & -0.5 & 1. & -0.5 & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & -0.5 & 1. & -0.5 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & -0.5 & 1. & -0.5 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 1. & -0.5 \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 1. \end{pmatrix}$$

$$\begin{pmatrix} 0.5 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ -0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & -0.5 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & -0.5 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & -0.5 & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 0. \end{pmatrix}$$

$$\begin{pmatrix} 0. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & -0.5 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & -0.5 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & -0.5 & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & -0.5 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & -0.5 \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\begin{pmatrix} 0. & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0.25 & -0.5 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.125 & 0.25 & -0.5 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0.0625 & -0.125 & 0.25 & -0.5 & 0. & 0. & 0. & 0. \\ 0. & -0.03125 & 0.0625 & -0.125 & 0.25 & -0.5 & 0. & 0. & 0. \\ 0. & 0.015625 & -0.03125 & 0.0625 & -0.125 & 0.25 & -0.5 & 0. & 0. \\ 0. & -0.0078125 & 0.015625 & -0.03125 & 0.0625 & -0.125 & 0.25 & -0.5 & 0. \\ 0. & 0.00390625 & -0.0078125 & 0.015625 & -0.03125 & 0.0625 & -0.125 & 0.25 & -0.5 \\ 0. & -0.00195313 & 0.00390625 & -0.0078125 & 0.015625 & -0.03125 & 0.0625 & -0.125 & 0.25 \end{pmatrix}$$

$$\{0.904508, 0.654508, 0.345492, 0.0954915, 8.31754 \times 10^{-9}, -8.31754 \times 10^{-9}, -3.3222 \times 10^{-16}, 0., 0.\}$$

0.904508

```
Clear[eq, f, ff]
Do[eq[i] = Reduce[sys[[i]], xx[i]], {i, 1, n - 1}];
Do[ff[i] = eq[i][[2]], {i, 1, n - 1}];
f = Table[ff[i], {i, 1, n - 1}]
```

$$\{0.5 + 0.5\,xx[2],\ 0. + 0.5\,xx[1] + 0.5\,xx[3],\ 0. + 0.5\,xx[2] + 0.5\,xx[4],\ 0. + 0.5\,xx[3] + 0.5\,xx[5],\ 0. + 0.5\,xx[4] + 0.5\,xx[6],\ 0. + 0.5\,xx[5] + 0.5\,xx[7],\ 0. + 0.5\,xx[6] + 0.5\,xx[8],\ 0. + 0.5\,xx[7] + 0.5\,xx[9],\ 0. + 0.5\,xx[8]\}$$

```
xk[0] = Table[0.0, {i, 1, n - 1}]
kmax = 20; tol = 1.0 × 10^(-4);
k = 0;
While[k < kmax,
 Do[
  sysold = Table[xx[j] == xk[k][[j]], {j, i + 1, n - 1}];
  sysnew = Table[xx[j] == xtemp[j], {j, 1, i - 1}];
  sys1 = Join[sysnew, sysold];
  sol1 = Solve[sys1, x][[1]];
  (*Print[i,sysold,sysnew,sys1,sol1];*)
  xtemp[i] = f[[i]] /. sol1 // N;
  , {i, 1, n}];
 xk[k + 1] = Table[xtemp[i], {i, 1, n - 1}];
 norm = Max[Abs[xk[k + 1] - xk[k]]]; Print[k, "  ", norm]; If[norm ≤ tol, Break[]];
 k++]
Print[xk[k]]
```

$$\{0., 0., 0., 0., 0., 0., 0., 0., 0.\}$$

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

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General::stop : Further output of Solve::svars will be suppressed during this calculation. ≫

Part::partw : Part 10 of {0.5 + 0.5 xx[2], ≪7≫, 0. + 0.5 xx[8]} does not exist. ≫

Part::partw :
Part 10 of {0.625, 0.3125, 0.15625, ≪3≫, 0.00976563, 0.00488281, 0.00195313} does not exist. ≫

```
0  0.5
```

Part::partw : Part 10 of {0.5 + 0.5 xx[2], ≪7≫, 0. + 0.5 xx[8]} does not exist. ≫

General::stop : Further output of Part::partw will be suppressed during this calculation. ≫

```
1  0.125

2  0.078125

3  0.0546875

4  0.0439453

5  0.0362549

6  0.0305481

7  0.0269165

8  0.0238819

9  0.0212955

10  0.0190611

11  0.0171093

12  0.0153892

13  0.0138632

14  0.0125024

15  0.0112843

16  0.0101909

17  0.00920737

18  0.00832136

19  0.00752229

{0.873087, 0.751336, 0.636325, 0.528836, 0.428858, 0.335666, 0.24796, 0.164043, 0.0820217}
```

```
Clear[eq, f, ff]
Do[eq[i] = Reduce[sys[[i]], xx[i]], {i, 1, n - 1}];
Do[ff[i] = eq[i][[2]], {i, 1, n - 1}];
f = Table[ff[i], {i, 1, n - 1}]
```

```
{0.5 + 0.5 xx[2], 0. + 0.5 xx[1] + 0.5 xx[3], 0. + 0.5 xx[2] + 0.5 xx[4],
 0. + 0.5 xx[3] + 0.5 xx[5], 0. + 0.5 xx[4] + 0.5 xx[6], 0. + 0.5 xx[5] + 0.5 xx[7],
 0. + 0.5 xx[6] + 0.5 xx[8], 0. + 0.5 xx[7] + 0.5 xx[9], 0. + 0.5 xx[8]}
```

```
xk[0] = Table[0.0, {i, 1, n - 1}]
kmax = 20; tol = 1.0 × 10^(-4);
ω = ω0;
k = 0;
While[k < kmax,
 Do[
  sysold = Table[xx[j] == xk[k][[j]], {j, i + 1, n - 1}];
  sysnew = Table[xx[j] == xtemp[j], {j, 1, i - 1}];
  sys1 = Join[sysnew, sysold];
  sol1 = Solve[sys1, x][[1]];
  (*Print[i,sysold,sysnew,sys1,sol1];*)
  xtemp[i] = ω * (f[[i]] /. sol1 // N) + (1. - ω) * xk[k][[i]];
  , {i, 1, n - 1}];
 xk[k + 1] = Table[xtemp[i], {i, 1, n - 1}];
 norm = Max[Abs[xk[k + 1] - xk[k]]]; Print[k, "  ", norm]; If[norm ≤ tol, Break[]];
 k++]
Print[xk[k]]
```

```
{0., 0., 0., 0., 0., 0., 0., 0., 0.}
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

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General::stop : Further output of Solve::svars will be suppressed during this calculation. ≫

```
0  0.763932

1  0.0759194

2  0.0548831

3  0.0427713

4  0.0347758

5  0.0271975

6  0.0196206

7  0.0124116

8  0.00582036

9  0.0018231

10  0.00185922

11  0.000975587

12  0.000547683

13  0.00029625

14  0.000151132

15  0.0000909228

{0.899871, 0.799814, 0.699798, 0.599809, 0.499862, 0.399902, 0.299938, 0.199966, 0.0999867}
```