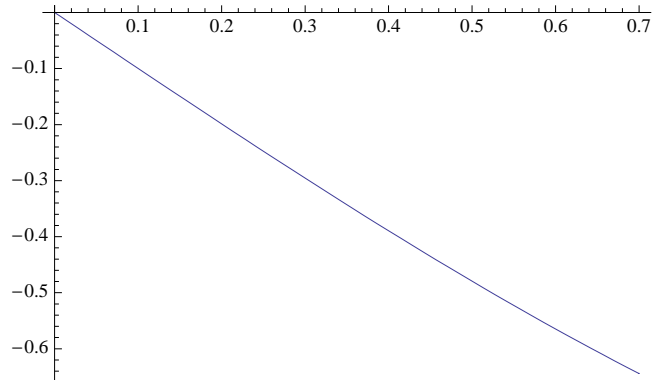


(** ESERCIZIO 4.1.2,3 **)

```
f[x_] := Sin[x];  
x[0] = 0.5; x[1] = 0.6; x[2] = 0.7; h = 0.1;  
y[0] = 0.4794; y[1] = 0.5646; y[2] = 0.6442;  
Plot[f''[x], {x, 0, 0.7}]  
Print["Derivate"]  
dy[0] = (y[1] - y[0]) / (x[1] - x[0])  
dyf[1] = (y[2] - y[1]) / (x[2] - x[1])  
dyb[1] = (y[1] - y[0]) / (x[1] - x[0])  
dy[2] = (y[2] - y[1]) / (x[2] - x[1])
```



Derivate

0.852

0.796

0.852

0.796

```
Print["Errori"]  
e[0] = Abs[dy[0] - f''[x[0]]]  
ef[1] = Abs[dyf[1] - f''[x[1]]]  
eb[1] = Abs[dyb[1] - f''[x[1]]]  
e[2] = Abs[dy[2] - f''[x[2]]]  
Print["Err. bound"]  
b[0] = Abs[h / 2] * Abs[f''[x[1]]]  
b[1] = Abs[h / 2] * Abs[f''[x[2]]]  
b[2] = Abs[h / 2] * Abs[f''[x[2]]]
```

Errori

0.0255826

0.0293356

0.0266644

0.0311578

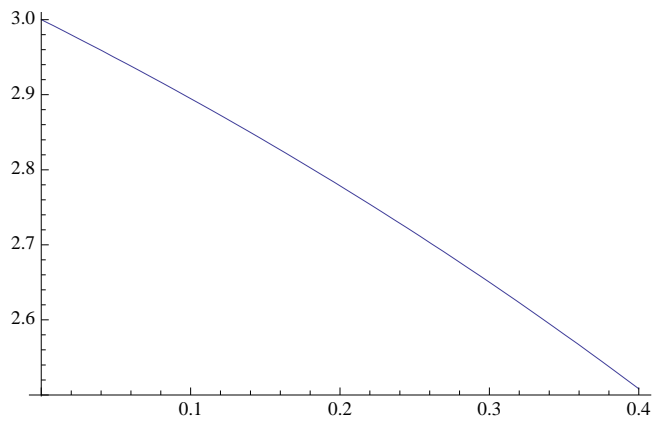
Err. bound

0.0282321

0.0322109

0.0322109

```
f[x_] := Exp[x] - 2 x^2 + 3 x - 1;  
Plot[Abs[f''[x]], {x, 0, 0.4}]
```



```
x[0] = 0.0; x[1] = 0.2; x[2] = 0.4; h = 0.2;
```

```
y[0] = f[x[0]]
```

```
y[1] = f[x[1]]
```

```
y[2] = f[x[2]]
```

```
0.
```

```
0.741403
```

```
1.37182
```

```

Print["Derivate"]
dy[0] = (y[1] - y[0]) / (x[1] - x[0])
dyf[1] = (y[2] - y[1]) / (x[2] - x[1])
dyb[1] = (y[1] - y[0]) / (x[1] - x[0])
dy[2] = (y[2] - y[1]) / (x[2] - x[1])
Print["Errori"]
e[0] = Abs[dy[0] - f'[x[0]]]
ef[1] = Abs[dyf[1] - f'[x[1]]]
eb[1] = Abs[dyb[1] - f'[x[1]]]
e[2] = Abs[dy[2] - f'[x[2]]]
Print["Err. bound"]
b[0] = Abs[h/2] * Abs[f''[x[0]]]
b[1] = Abs[h/2] * Abs[f''[x[1]]]
b[2] = Abs[h/2] * Abs[f''[x[1]]]

```

Derivate

```

3.70701
3.15211
3.70701
3.15211

```

Errori

```

0.292986
0.269293
0.285611
0.260285

```

Err. bound

```

0.3
0.27786
0.27786

```

(** ESERCIZIO 4.4.1 e 2 **)

```

(* (a) *)
f[x_] := x * Log[x];
a = 1.; b = 2.;
true1 = Integrate[f[x], x]
true = Integrate[f[x], {x, a, b}] // N

```

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \text{Log}[x]$$

```

0.636294

```

```

x[0] = a;
n = 4;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];

```

```
true = 0.636294
```

```
trap = 0.6399
```

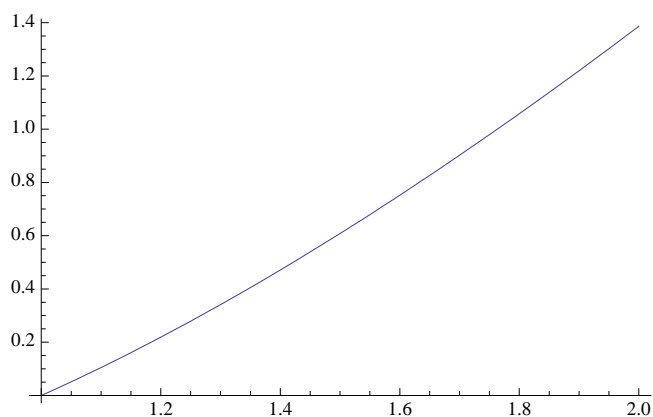
```
error = 0.00360612
```

```
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];
Plot[f[x], {x, a, b}]
```

```
true = 0.636294
```

```
simps = 0.63631
```

```
error = 0.0000154701
```



```
(* (e) *)
Clear[x, f];
f[x_] := Exp[2 x] * Sin[3 x];
a = 0.; b = 2.;
true1 = Integrate[f[x], x]
true = Integrate[f[x], {x, a, b}] // N
```

$$\frac{1}{13} e^{2x} (-3 \cos[3x] + 2 \sin[3x])$$

```
-14.214
```

```
x[0] = a;
n = 8;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];
```

```
true = -14.214
```

```
trap = -13.576
```

```
error = 0.637998
```

```

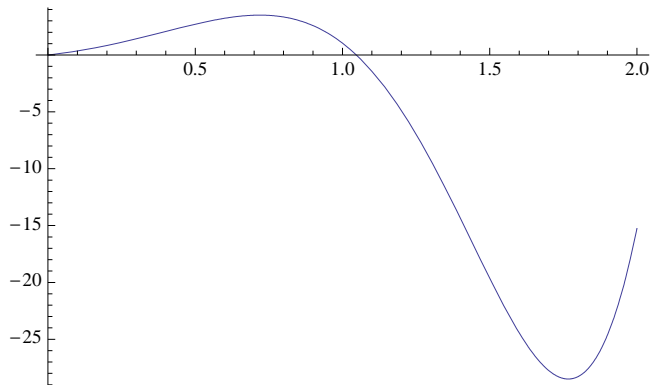
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];
Plot[f[x], {x, a, b}]

```

true = -14.214

simps = -14.1833

error = 0.0306356



(** ESERCIZIO 4.4.7 **)

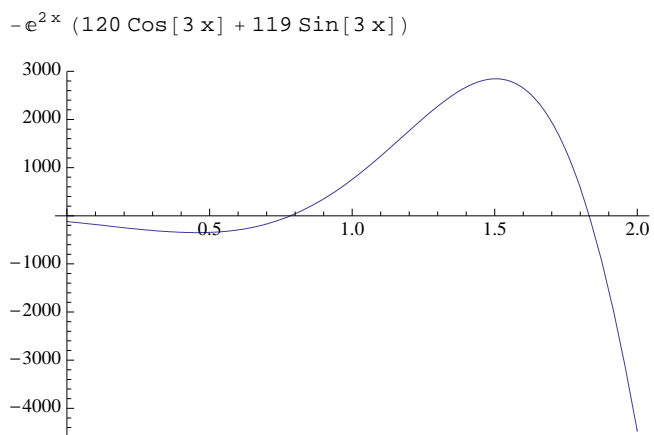
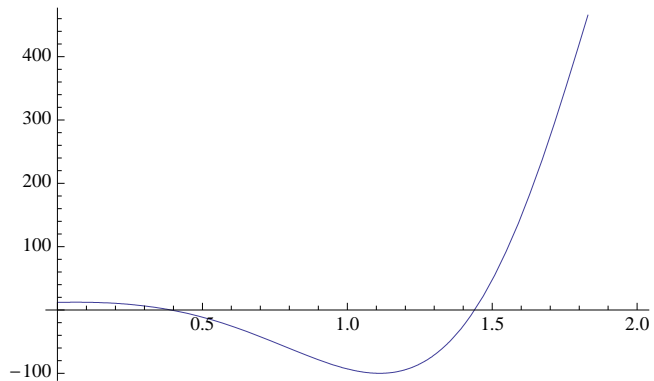
```

Clear[x, f];
f[x_] := Exp[2 x] * Sin[3 x];
a = 0.; b = 2.;
true1 = Integrate[f[x], x];
true = Integrate[f[x], {x, a, b}] // N
-14.214

```

```
(* Stima teorica *)
Simplify[f''[x]]
Plot[f''[x], {x, a, b}]
Simplify[f''''[x]]
Plot[f''''[x], {x, a, b}]

 $e^{2x} (12 \cos[3x] - 5 \sin[3x])$ 
```



```
(* Trapezi *)
Clear[h];
tol = 0.0001;
errbound = (b - a) / 12 * h^2 * f''[b]
hbound = Sqrt[12. * tol / ((b - a) * f''[b])]
nbound = (b - a) / hbound
```

$117.56 h^2$

0.000922296

2168.5

```
(* Simpson *)
Clear[h];
tol = 0.0001;
errbound = Abs[(b - a) / 180 * h^4 * f''''[b]]
hbound = Sqrt[Sqrt[Abs[180. * tol / ((b - a) * f''''[b])]]]
nbound = (b - a) / hbound
```

$49.7268 \text{ Abs}[h]^4$

0.0376576

53.1102

```
(* Calcolo numerico *)
(* Trapezi *)
x[0] = a;
n = 800;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];
```

true = -14.214

trap = -14.2139

error = 0.000064458

```
(* Simpson *)
x[0] = a;
n = 34;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];
```

true = -14.214

simps = -14.2139

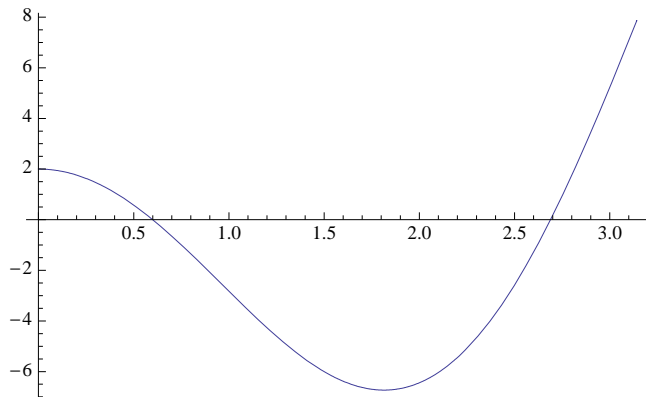
error = 0.0000783569

(** ESERCIZIO 4.4.8 **)

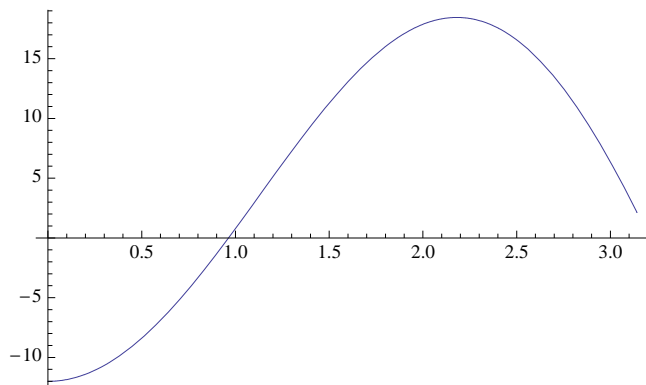
```
Clear[x, f];
f[x_] := x^2 * Cos[x];
a = 0.; b = Pi;
true1 = Integrate[f[x], x];
true = Integrate[f[x], {x, a, b}] // N
-6.28319
```

```
(* Stima teorica *)
Simplify[f''[x]]
Plot[f''[x], {x, a, b}]
Simplify[f''''[x]]
Plot[f''''[x], {x, a, b}]

-(-2 + x2) Cos[x] - 4 x Sin[x]
```



```
(-12 + x2) Cos[x] + 8 x Sin[x]
```



```
(* Trapezi *)
Clear[h];
tol = 0.0001;
errbound = (b - a) / 12 * h2 * f''[b]
hbound = Sqrt[12. * tol / ((b - a) * f''[b])]
nbound = (b - a) / hbound
```

2.06026 h²

0.00696689

450.932

```
(* Simpson *)
Clear[h];
tol = 0.0001;
errbound = Abs[(b - a) / 180 * h4 * f''''[b]]
hbound = Sqrt[Sqrt[Abs[180. * tol / ((b - a) * f''''[b])]]]
nbound = (b - a) / hbound
```

0.0371824 Abs[h]⁴

0.227728

13.7954


```

(* Calcolo numerico *)
(* Trapezi *)
x[0] = a;
n = 400;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];

```

true = -6.28319

trap = -6.28322

error = 0.0000322983

```

(* Simpson *)
x[0] = a;
n = 20;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];

```

true = -6.28319

simps = -6.28312

error = 0.0000640674