

(* PROBLEMI AL CONTORNO *)

(* ESERCIZIO 11.3.1 *)

```
In[1]:= Clear[a, b, p, q, r, w, x, aa]
fp[x_] := 0;
fq[x_] := 4;
fr[x_] := -4 x;
a = 0.;
b = 1.;
b = 4.;
α = 0.;
β = 2.;
xrange = {a, b + 0.2};
eqdiff = y'[x] == fp[x] * y'[x] + fq[x] * y[x] + fr[x];
soldiff = DSolve[{eqdiff, y[a] == α, y[b] == β}, y[x], x];
ysol[x_] = y[x] /. soldiff
```

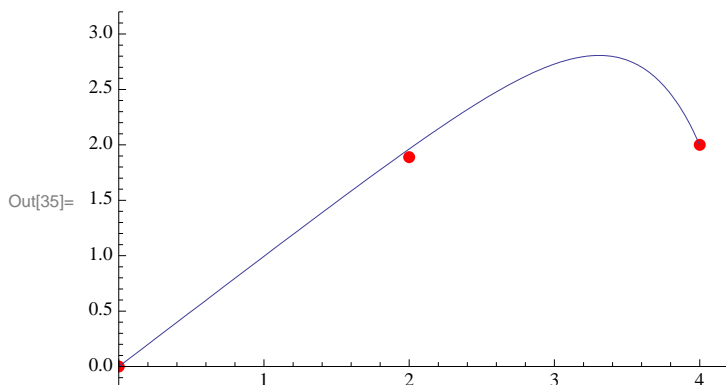
Out[13]= $\{e^{-2x} (0.000670925 - 0.000670925 e^{4x} + e^{2x} x)\}$

```
In[14]:= n = 2;
h = (b - a) / n;
x[1] = a;
x[n + 1] = b;
ww = Table[w[i], {i, 1, n + 1}];
Do[x[i + 1] = x[i] + h, {i, 1, n}];
Do[p[i] = fp[x[i]]; q[i] = fq[x[i]]; r[i] = fr[x[i]], {i, 1, n + 1}];
aa = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
aa[[1, 1]] = 1.;
aa[[n + 1, n + 1]] = 1.;
MatrixForm[aa];
Do[aa[[i, i - 1]] = 2. + h * p[i];
aa[[i, i + 1]] = 2. - h * p[i]; aa[[i, i]] = -2. * (2. + h^2 * q[i]), {i, 2, n}];
MatrixForm[aa]
bb = Table[2. * h^2 * r[i], {i, 1, n + 1}];
bb[[1]] = α;
bb[[n + 1]] = β;
MatrixForm[bb]
```

Out[26]/MatrixForm=
$$\begin{pmatrix} 1. & 0 & 0 \\ 2. & -36. & 2. \\ 0 & 0 & 1. \end{pmatrix}$$

Out[30]/MatrixForm=
$$\begin{pmatrix} 0. \\ -64. \\ 2. \end{pmatrix}$$

```
In[31]:= sol = Solve[Dot[aa, ww] == bb, ww][[1]];
fsol = Table[{x[i], w[i] /. sol}, {i, 1, n + 1}];
p11 = ListPlot[fsol, PlotStyle -> {PointSize[0.02], RGBColor[1, 0, 0]};
p12 = Plot[ysol[x], {x, a, b}];
Show[p11, p12, PlotRange -> {xrange, {-0.2, 3.2}}]
```



```
In[36]:= (* ESERCIZIO 11.3.2 *)
Clear[a, b, p, q, r, w, x, aa]
fp[x_] := 1;
fq[x_] := 2;
fr[x_] := Cos[x];
a = 0.;
b = 0.5 * Pi;
α = -0.3;
β = -0.1;
eqdiff = y''[x] == fp[x] * y'[x] + fq[x] * y[x] + fr[x];
soldiff = DSolve[{eqdiff, y[a] == α, y[b] == β}, y[x], x];
ysol[x_] = y[x] /. soldiff
```

```
Out[46]= {e^{-x} (5.4804 × 10^{-17} + 7.07106 × 10^{-19} e^{3x} - 0.3 e^x Cos[x] - 0.1 e^x Sin[x])}
```

```
In[47]:= n = 8;
h = (b - a) / n;
x[1] = a;
x[n + 1] = b;
ww = Table[w[i], {i, 1, n + 1}];
Do[x[i + 1] = x[i] + h, {i, 1, n}];
Do[p[i] = fp[x[i]]; q[i] = fq[x[i]]; r[i] = fr[x[i]], {i, 1, n + 1}];
aa = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
aa[[1, 1]] = 1.;
aa[[n + 1, n + 1]] = 1.;
MatrixForm[aa];
Do[aa[[i, i - 1]] = 2. + h * p[i];
aa[[i, i + 1]] = 2. - h * p[i]; aa[[i, i]] = -2. * (2. + h^2 * q[i]), {i, 2, n}];
MatrixForm[aa]
bb = Table[2. * h^2 * r[i], {i, 1, n + 1}];
bb[[1]] = α;
bb[[n + 1]] = β;
MatrixForm[bb]
```

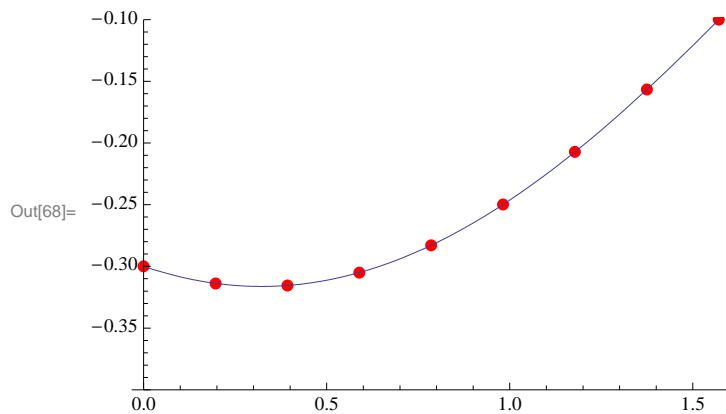
```
Out[59]/MatrixForm=
```

$$\begin{pmatrix} 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.19635 & -4.15421 & 1.80365 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.19635 & -4.15421 & 1.80365 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.19635 & -4.15421 & 1.80365 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.19635 & -4.15421 & 1.80365 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.19635 & -4.15421 & 1.80365 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.19635 & -4.15421 & 1.80365 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.19635 & -4.15421 & 1.80365 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. \end{pmatrix}$$

```
Out[63]/MatrixForm=
```

$$\begin{pmatrix} -0.3 \\ 0.0756247 \\ 0.0712369 \\ 0.0641115 \\ 0.0545224 \\ 0.042838 \\ 0.0295073 \\ 0.0150427 \\ -0.1 \end{pmatrix}$$

```
In[64]:= sol = Solve[Dot[aa, ww] == bb, ww][[1]];
fsol = Table[{x[i], w[i] /. sol}, {i, 1, n + 1}];
pl1 = ListPlot[fsol, PlotStyle -> {PointSize[0.02], RGBColor[1, 0, 0]}];
pl2 = Plot[ysol[x], {x, a, b}];
Show[pl1, pl2, PlotRange -> {-0.4, -0.1}]
```



```
In[69]:= (* ESERCIZIO LIBERO *)
Clear[a, b, p, q, r, w, x, aa]
fp[x_] := 0;
fq[x_] := -9;
fr[x_] := 0;
a = 0.;
b = 1.;
α = 1.;
β = 0.5;
xrange = {a, b};
yrange = {-20, 20};
eqdiff = y''[x] == fp[x] * y'[x] + fq[x] * y[x] + fr[x];
soldiff = DSolve[{eqdiff, y[a] == α, y[b] == β}, y[x], x];
ysol[x_] = y[x] /. soldiff
```

Out[81]= {1. Cos[3 x] + 10.5583 Sin[3 x]}

```

In[82]:= n = 4;
h = (b - a) / n;
x[1] = a;
x[n + 1] = b;
ww = Table[w[i], {i, 1, n + 1}];
Do[x[i + 1] = x[i] + h, {i, 1, n}];
Do[p[i] = fp[x[i]]; q[i] = fq[x[i]]; r[i] = fr[x[i]], {i, 1, n + 1}];
aa = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
aa[[1, 1]] = 1.;
aa[[n + 1, n + 1]] = 1.;
MatrixForm[aa];
Do[aa[[i, i - 1]] = 2. + h * p[i];
  aa[[i, i + 1]] = 2. - h * p[i]; aa[[i, i]] = -2. * (2. + h^2 * q[i]), {i, 2, n}];
MatrixForm[aa]
bb = Table[2. * h^2 * r[i], {i, 1, n + 1}];
bb[[1]] =  $\alpha$ ;
bb[[n + 1]] =  $\beta$ ;
MatrixForm[bb]

```

```

Out[94]//MatrixForm=

$$\begin{pmatrix} 1. & 0 & 0 & 0 & 0 \\ 2. & -2.875 & 2. & 0 & 0 \\ 0 & 2. & -2.875 & 2. & 0 \\ 0 & 0 & 2. & -2.875 & 2. \\ 0 & 0 & 0 & 0 & 1. \end{pmatrix}$$


```

```

Out[98]//MatrixForm=

```

```

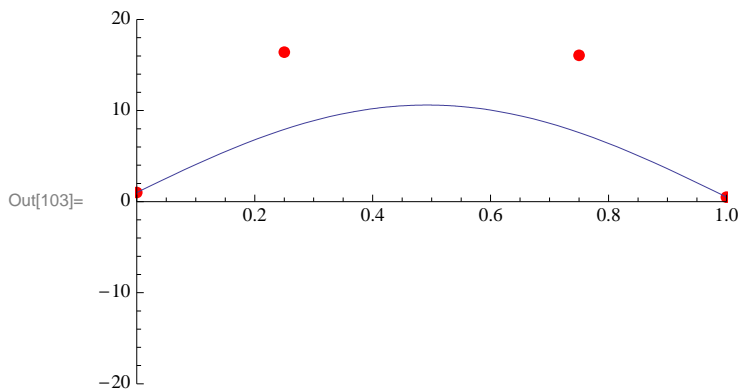

$$\begin{pmatrix} 1. \\ 0 \\ 0 \\ 0 \\ 0.5 \end{pmatrix}$$


```

```

In[99]:= sol = Solve[Dot[aa, ww] == bb, ww][[1]];
fsol = Table[{x[i], w[i] /. sol}, {i, 1, n + 1}];
p11 = ListPlot[fsol, PlotStyle -> {PointSize[0.02], RGBColor[1, 0, 0]};
p12 = Plot[ysol[x], {x, a, b}];
Show[p11, p12, PlotRange -> {xrange, yrange}]

```



```
In[104]:= (* ESERCIZIO 11.3.3b *)
Clear[a, b, p, q, r, w, x, aa]
fp[x_] := -4./x;
fq[x_] := 2./x^2;
fr[x_] := -(2./x^2)*Log[x];
a = 1.;
b = 2.;
α = -0.5;
β = Log[2.];
xrange = {a, b};
yrange = {-0.5, 1};
eqdiff = y'[x] == fp[x]*y'[x] + fq[x]*y[x] + fr[x];
soldiff = DSolve[{eqdiff, y[a] == α, y[b] == β}, y[x], x];
ysol[x_] = y[x] /. soldiff
```

```
Out[116]= 
$$\left\{ \frac{(-1.04353 + 0. i) + (1.5 + 0. i) x^{3.56155} - (0.956473 + 0. i) x^{4.12311} + (1. + 0. i) x^{3.56155} \text{Log}[x]}{x^{3.56155}} \right\}$$

```

```
In[117]:= n = 2;
h = (b - a) / n;
x[1] = a;
x[n + 1] = b;
ww = Table[w[i], {i, 1, n + 1}];
Do[x[i + 1] = x[i] + h, {i, 1, n}];
Do[p[i] = fp[x[i]]; q[i] = fq[x[i]]; r[i] = fr[x[i]], {i, 1, n + 1}];
aa = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
aa[[1, 1]] = 1.;
aa[[n + 1, n + 1]] = 1.;
MatrixForm[aa];
Do[aa[[i, i - 1]] = 2. + h*p[i];
aa[[i, i + 1]] = 2. - h*p[i]; aa[[i, i]] = -2.* (2. + h^2*q[i]), {i, 2, n}];
MatrixForm[aa];
bb = Table[2.*h^2*r[i], {i, 1, n + 1}];
bb[[1]] = α;
bb[[n + 1]] = β;
MatrixForm[bb]
```

```
Out[129]/MatrixForm=

$$\begin{pmatrix} 1. & 0 & 0 \\ 0.666667 & -4.44444 & 3.33333 \\ 0 & 0 & 1. \end{pmatrix}$$

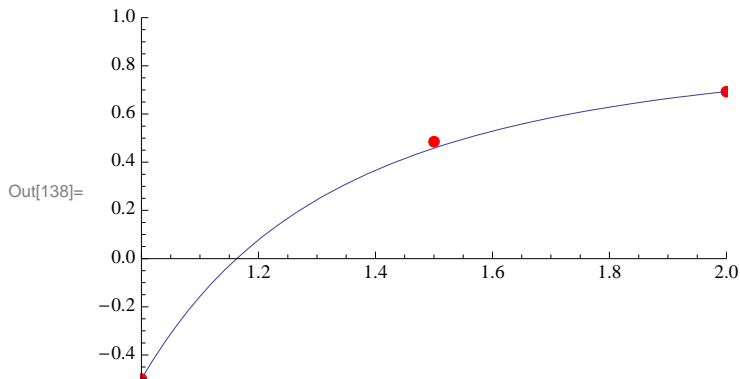
```

```
Out[133]/MatrixForm=

$$\begin{pmatrix} -0.5 \\ -0.180207 \\ 0.693147 \end{pmatrix}$$

```

```
In[134]:= sol = Solve[Dot[aa, ww] == bb, ww][[1]];
fsol = Table[{x[i], w[i] /. sol}, {i, 1, n + 1}];
p11 = ListPlot[fsol, PlotStyle -> {PointSize[0.02], RGBColor[1, 0, 0]};
p12 = Plot[ysol[x], {x, a, b}];
Show[p11, p12, PlotRange -> {xrange, yrange}]
```



```
In[289]:= (* ESERCIZIO 11.3.3c *)
Clear[a, b, p, q, r, w, x, aa]
fp[x_] := -(x + 1);
fq[x_] := 2;
fr[x_] := (1 - x^2) * Exp[-x];
a = 0;
b = 1;
α = -1;
β = 0;
xrange = {a - 0.1, b + 0.1};
yrange = {-1.1, 0.1};
eqdiff = y'[x] == fp[x] * y'[x] + fq[x] * y[x] + fr[x];
soldiff = NDSolve[{eqdiff, y[a] == α, y[b] == β}, y[x], x];
```

```
In[301]:= ysol[x_] = y[x] /. soldiff;
```

```
In[302]:= n = 2;
h = (b - a) / n;
x[1] = a;
x[n + 1] = b;
ww = Table[w[i], {i, 1, n + 1}];
Do[x[i + 1] = x[i] + h, {i, 1, n}];
Do[p[i] = fp[x[i]]; q[i] = fq[x[i]]; r[i] = fr[x[i]], {i, 1, n + 1}];
aa = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
aa[[1, 1]] = 1.;
aa[[n + 1, n + 1]] = 1.;
MatrixForm[aa];
Do[aa[[i, i - 1]] = 2. + h * p[i];
aa[[i, i + 1]] = 2. - h * p[i]; aa[[i, i]] = -2. * (2. + h^2 * q[i]), {i, 2, n}];
MatrixForm[aa];
bb = Table[2. * h^2 * r[i], {i, 1, n + 1}];
bb[[1]] = α;
bb[[n + 1]] = β;
MatrixForm[bb]
```

Out[314]/MatrixForm=

$$\begin{pmatrix} 1. & 0 & 0 \\ 1.25 & -5. & 2.75 \\ 0 & 0 & 1. \end{pmatrix}$$

Out[318]/MatrixForm=

$$\begin{pmatrix} -1 \\ 0.227449 \\ 0 \end{pmatrix}$$

```
In[319]:= sol = Solve[Dot[aa, ww] == bb, ww][[1]];
fsol = Table[{x[i], w[i] /. sol}, {i, 1, n + 1}];
p11 = ListPlot[fsol, PlotStyle -> {PointSize[0.02], RGBColor[1, 0, 0]};
p12 = Plot[ysol[x], {x, a, b}];
Show[p11, p12, PlotRange -> {xrange, yrange}]
```

