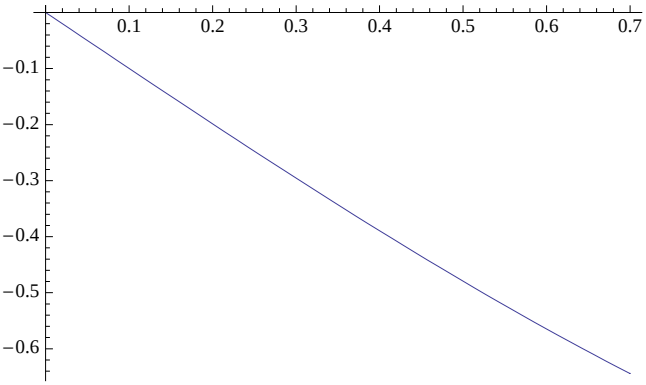
```
(** ESERCIZIO 4.1.2,3 **)
```

In[1]:=
```
f[x_] := Sin[x];
x[0] = 0.5; x[1] = 0.6; x[2] = 0.7; h = 0.1;
y[0] = 0.4794; y[1] = 0.5646; y[2] = 0.6442;
Plot[f''[x], {x, 0, 0.7}]
Print["Derivate"]
dy[0] = (y[1] - y[0]) / (x[1] - x[0])
dyf[1] = (y[2] - y[1]) / (x[2] - x[1])
dyb[1] = (y[1] - y[0]) / (x[1] - x[0])
dy[2] = (y[2] - y[1]) / (x[2] - x[1])
```

Out[4]=




```
Derivate
```

Out[6]= 0.852

Out[7]= 0.796

Out[8]= 0.852

Out[9]= 0.796

In[10]:=
```
Print["Errori"]
e[0] = Abs[dy[0] - f'[x[0]]]
ef[1] = Abs[dyf[1] - f'[x[1]]]
eb[1] = Abs[dyb[1] - f'[x[1]]]
e[2] = Abs[dy[2] - f'[x[2]]]
Print["Err. bound"]
b[0] = Abs[h / 2] * Abs[f''[x[1]]]
b[1] = Abs[h / 2] * Abs[f''[x[2]]]
b[2] = Abs[h / 2] * Abs[f''[x[2]]]
```

```
Errori
```

Out[11]= 0.0255826

Out[12]= 0.0293356

Out[13]= 0.0266644

Out[14]= 0.0311578

```
Err. bound
```

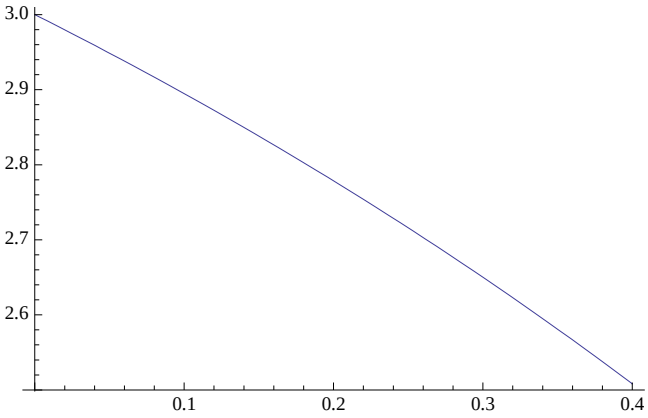
Out[16]= 0.0282321

Out[17]= 0.0322109

Out[18]= 0.0322109

In[19]:=
```
f[x_] := Exp[x] - 2 x^2 + 3 x - 1;
Plot[Abs[f''[x]], {x, 0, 0.4}]
```

Out[20]=



In[21]:=
```
x[0] = 0.0; x[1] = 0.2; x[2] = 0.4; h = 0.2;
y[0] = f[x[0]]
y[1] = f[x[1]]
y[2] = f[x[2]]
```

Out[22]= `0.`

Out[23]= `0.741403`

Out[24]= `1.37182`

In[25]:=
```
Print["Derivate"]
dy[0] = (y[1] - y[0]) / (x[1] - x[0])
dyf[1] = (y[2] - y[1]) / (x[2] - x[1])
dyb[1] = (y[1] - y[0]) / (x[1] - x[0])
dy[2] = (y[2] - y[1]) / (x[2] - x[1])
Print["Errori"]
e[0] = Abs[dy[0] - f'[x[0]]]
ef[1] = Abs[dyf[1] - f'[x[1]]]
eb[1] = Abs[dyb[1] - f'[x[1]]]
e[2] = Abs[dy[2] - f'[x[2]]]
Print["Err. bound"]
b[0] = Abs[h / 2] * Abs[f''[x[0]]]
b[1] = Abs[h / 2] * Abs[f''[x[1]]]
b[2] = Abs[h / 2] * Abs[f''[x[1]]]
```

```
Derivate
```

Out[26]= 3.70701

Out[27]= 3.15211

Out[28]= 3.70701

Out[29]= 3.15211

```
Errori
```

Out[31]= 0.292986

Out[32]= 0.269293

Out[33]= 0.285611

Out[34]= 0.260285

```
Err. bound
```

Out[36]= 0.3

Out[37]= 0.27786

Out[38]= 0.27786

```
(** ESERCIZIO 4.4.1 e 2 **)
```

In[39]:=
```
(* (a) *)
f[x_] := x * Log[x];
a = 1.; b = 2.;
true1 = Integrate[f[x], x]
true = Integrate[f[x], {x, a, b}] // N
```

Out[41]= $-\frac{x^2}{4} + \frac{1}{2} x^2 \text{Log}[x]$

Out[42]= 0.636294

In[43]:=
```
x[0] = a;
n = 4;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];
```

```
true = 0.636294

trap = 0.6399

error = 0.00360612
```

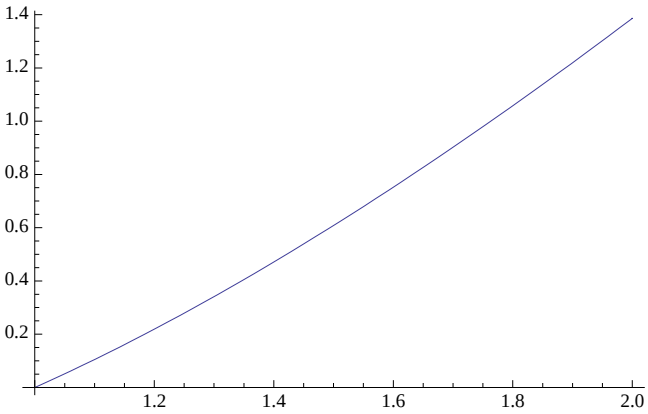
In[55]:=
```
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];
Plot[f[x], {x, a, b}]
```

```
true = 0.636294

simps = 0.63631

error = 0.0000154701
```

Out[65]=



In[66]:=
```
(* (e) *)
Clear[x, f];
f[x_] := Exp[2 x] * Sin[3 x];
a = 0.; b = 2.;
true1 = Integrate[f[x], x]
true = Integrate[f[x], {x, a, b}] // N
```

Out[69]= $\frac{1}{13} e^{2x} (-3 \cos[3x] + 2 \sin[3x])$

Out[70]= −14.214

In[71]:=
```
x[0] = a;
n = 8;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];
```

```
true = -14.214

trap = -13.576

error = 0.637998
```

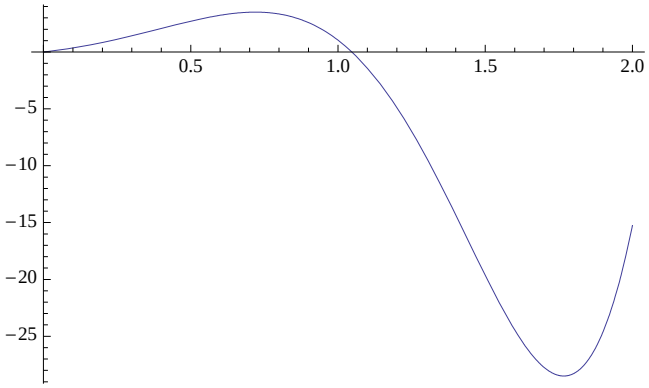
In[83]:=
```
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];
Plot[f[x], {x, a, b}]
```

```
true = -14.214

simps = -14.1833

error = 0.0306356
```

Out[93]=




```
(** ESERCIZIO 4.4.7 **)
```

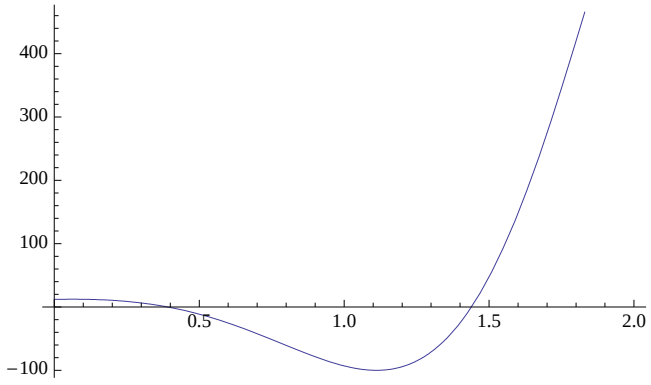
In[94]:=
```
Clear[x, f];
f[x_] := Exp[2 x] * Sin[3 x];
a = 0.; b = 2.;
true1 = Integrate[f[x], x];
true = Integrate[f[x], {x, a, b}] // N
```

Out[98]= −14.214

In[99]:=
```
(* Stima teorica *)
Simplify[f''[x]]
Plot[f''[x], {x, a, b}]
Simplify[f''''[x]]
Plot[f''''[x], {x, a, b}]
```

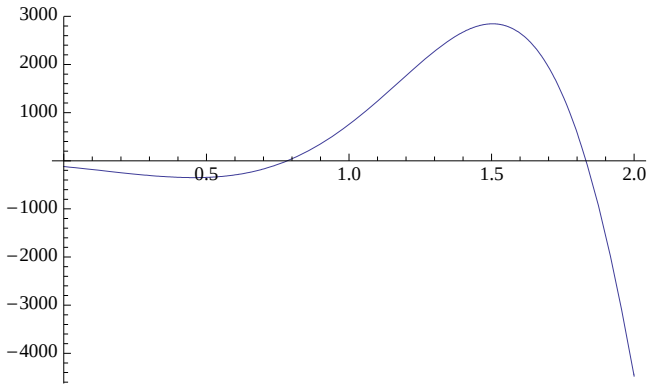
Out[99]= $e^{2x}(12\cos[3x] - 5\sin[3x])$

Out[100]=



Out[101]= $-e^{2x}(120\cos[3x] + 119\sin[3x])$

Out[102]=



In[103]:=
```
(* Trapezi *)
Clear[h];
tol = 0.0001;
errbound = (b - a) / 12 * h^2 * f''[b]
hbound = Sqrt[12. * tol / ((b - a) * f''[b])]
nbound = (b - a) / hbound
```

Out[105]= $117.56\,h^2$

Out[106]= 0.000922296

Out[107]= 2168.5

In[108]:=
```
(* Simpson *)
Clear[h];
tol = 0.0001;
errbound = Abs[(b - a) / 180 * h^4 * f''''[b]]
hbound = Sqrt[Sqrt[Abs[180. * tol / ((b - a) * f''''[b])]]]
nbound = (b - a) / hbound
```

Out[110]= $49.7268\,\text{Abs}[h]^4$

Out[111]= 0.0376576

Out[112]= 53.1102

In[113]:=
```
(* Calcolo numerico *)
(* Trapezi *)
x[0] = a;
n = 800;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];
```

```
true = -14.214

trap = -14.2139

error = 0.000064458
```

In[125]:=
```
(* Simpson *)
x[0] = a;
n = 34;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];
```

```
true = -14.214

simps = -14.2139

error = 0.0000783569
```

```
(** ESERCIZIO 4.4.8 **)
```

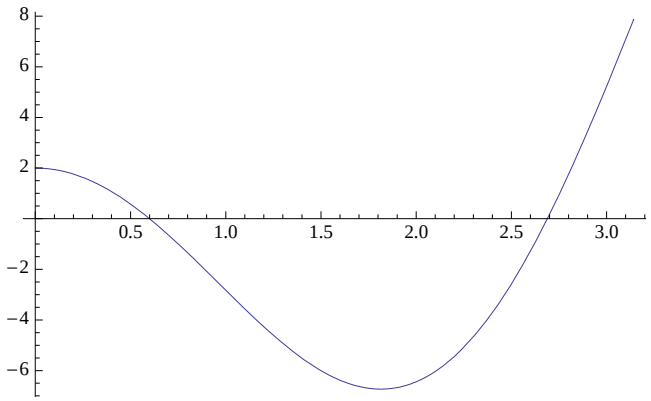
In[140]:=
```
Clear[x, f];
f[x_] := x^2 * Cos[x];
a = 0.; b = Pi;
true1 = Integrate[f[x], x];
true = Integrate[f[x], {x, a, b}] // N
```

Out[144]=
```
-6.28319
```

In[145]:=
```
(* Stima teorica *)
Simplify[f''[x]]
Plot[f''[x], {x, a, b}]
Simplify[f''''[x]]
Plot[f''''[x], {x, a, b}]
```

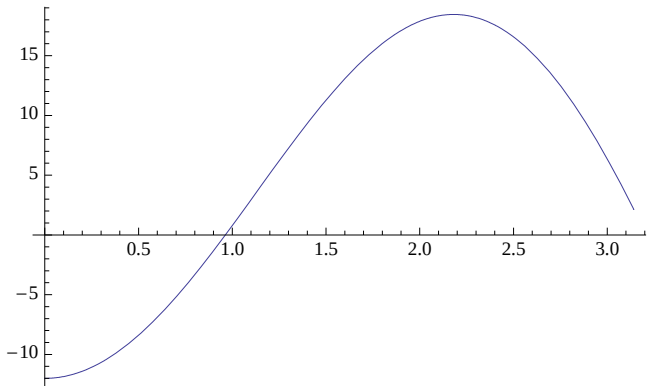
Out[145]= $-\left(-2+x^2\right)\,\text{Cos}[x]-4\,x\,\text{Sin}[x]$

Out[146]=



Out[147]= $\left(-12+x^2\right)\,\text{Cos}[x]+8\,x\,\text{Sin}[x]$

Out[148]=



In[149]:=
```
(* Trapezi *)
Clear[h];
tol = 0.0001;
errbound = (b - a) / 12 * h^2 * f''[b]
hbound = Sqrt[12. * tol / ((b - a) * f''[b])]
nbound = (b - a) / hbound
```

Out[151]= $2.06026\,h^2$

Out[152]= 0.00696689

Out[153]= 450.932

In[154]:=
```
(* Simpson *)
Clear[h];
tol = 0.0001;
errbound = Abs[(b - a) / 180 * h^4 * f''''[b]]
hbound = Sqrt[Sqrt[Abs[180. * tol / ((b - a) * f''''[b])]]]
nbound = (b - a) / hbound
```

Out[156]= $0.0371824\,\text{Abs}[h]^4$

Out[157]= 0.227728

Out[158]= 13.7954

In[159]:=
```
(* Calcolo numerico *)
(* Trapezi *)
x[0] = a;
n = 400;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
sum = 0.0;
Do[sum += f[x[i]], {i, 1, n - 1}];
trap = 0.5 * h * (f[a] + 2.0 * sum + f[b]);
err = Abs[trap - true];
Print["true = ", true];
Print["trap = ", trap];
Print["error = ", err];
```

```
true = -6.28319

trap = -6.28322

error = 0.0000322983
```

In[171]:=
```
(* Simpson *)
x[0] = a;
n = 20;
x[n] = b;
h = (b - a) / n;
Do[x[i + 1] = x[i] + h, {i, 0, n - 2}];
w[0] = 1.; w[n] = 1.;
Do[w[i] = 4., {i, 1, n - 1, 2}]
Do[w[i] = 2., {i, 2, n - 2, 2}]
sum = 0.0;
Do[sum += w[i] * f[x[i]], {i, 0, n}];
simps = h * sum / 3.;
err = Abs[simps - true];
Print["true = ", true];
Print["simps = ", simps];
Print["error = ", err];
```

```
true = -6.28319

simps = -6.28312

error = 0.0000640674
```