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(* FRANCESCA CAMPI *)
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(* 1 *)
F[x_, y_] := {x * y, x^2 * y}
r[t_] := {Sqrt[1 + t^2], t}
```

In[13]:=
```
f[t_] = F[x, y] /. {x → r[t][[1]], y → r[t][[2]]}
fs[t_] = Dot[f[t], r'[t]]
```

Out[13]=
$$\left\{t \sqrt{1+t^2},\ t\left(1+t^2\right)\right\}$$

Out[14]=
$$t^2 + t\left(1+t^2\right)$$

In[15]:=
```
Integrate[fs[t], {t, -2, 2}]
```

Out[15]=
$$\frac{16}{3}$$

In[16]:=
```
(* 2 *)
F[x_, y_] := {2 x + 1, x * y}
```

In[30]:=
```
eq1 = x^2 + y^2 == 1;
eq2 = Sqrt[3] y == x + 1;
Solve[{eq1, eq2}, {x, y}]
```

Out[32]=
$$\left\{\{x \to -1,\ y \to 0\},\ \left\{x \to \frac{1}{2},\ y \to \frac{\sqrt{3}}{2}\right\}\right\}$$

In[34]:=
```
r1[t_] := {Cos[t], Sin[t]}
r2[t_] := {t, (t + 1) / Sqrt[3]}
```

In[40]:=
```
f1[t_] = F[x, y] /. {x → r1[t][[1]], y → r1[t][[2]]}
f2[t_] = F[x, y] /. {x → r2[t][[1]], y → r2[t][[2]]}
fs1[t_] = Dot[f1[t], r1'[t]]
fs2[t_] = Dot[f2[t], r2'[t]]
```

Out[40]=
$$\{1 + 2\,\mathrm{Cos}[t],\ \mathrm{Cos}[t]\,\mathrm{Sin}[t]\}$$

Out[41]=
$$\left\{1 + 2\,t,\ \frac{t\,(1+t)}{\sqrt{3}}\right\}$$

Out[42]=
$$\mathrm{Cos}[t]^2\,\mathrm{Sin}[t] - (1 + 2\,\mathrm{Cos}[t])\,\mathrm{Sin}[t]$$

Out[43]=
$$1 + 2\,t + \frac{1}{3}\,t\,(1+t)$$

In[48]:=
```
Integrate[fs1[t], {t, -π, π / 3}] + Integrate[fs2[t], {t, 1 / 2, -1}]
```

Out[48]=
$$-\frac{3}{8}$$