School "Variational and Geometric Methods in PDEs"
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"Nonlinear diffusion equations, Diffusion with nonlocal operators"

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Abstract

The study of nonlinear diffusion has been the object of research for decades both in the stationary and evolution (parabolic) setting). We will look at degenerate or singular nonlinear models like the porous medium or fast diffusion flows. We will present some recent results in the fast diffusion range.

Recently, research has focused on nonlinear elliptic and parabolic equations involving fractional Laplacian operators or other similar nonlocal operators. We present two models for flow in porous media including nonlocal (long-range) diffusion effects of such type. The first one is based on Darcys law and the pressure is related to the density by an inverse fractional Laplacian operator. We prove existence of solutions that propagate with finite speed, which is unexpected in fractional diffusion models. The model has also the very interesting property that mass preserving selfsimilar solutions can be found by solving an elliptic obstacle problem

The second model comes from statistical mechanics considerations, generalizes the well-known linear fractional heat equation, generates a nice nonlinear contractive semigroup, and has infinite speed of propagation for all powers of the nonlinearity.

Some references.

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