

• **Laura Abatangelo**

*"Positive harmonic functions in union of chambers domains"*

In domains which are formed by several different chambers (e.g. a finite sequence of cylinders) there exists a unique regular positive harmonic function which is zero on the boundary. This has an infinite energy at least at one hand of the chambers' sequence.

In such domains, whose peculiar feature is a sensitive change of geometry at the connecting points of two adjoining chambers, it seems reasonable that the solution's behavior is strictly related to the domain's geometry. From another point of view, the sudden change of geometry passing from a chamber to another affects the asymptotic behavior of the solution. The subject which rules this relation is a "transfer" operator which is in fact a suitable basis changing operator.

This is a joint work with Susanna Terracini.

• **Daniele Castorina** - Universitat Autònoma de Barcelona

*"Spectral theory for linearized  $p$ -Laplace equations"*

We continue and completely set up the spectral theory initiated in [CES1] for the linearized operator arising from  $\Delta_p u + f(u) = 0$ . We establish existence and variational characterization of all the eigenvalues, and by a weak Harnack inequality we deduce Hölder continuity for the corresponding eigenfunctions, this regularity being sharp. The Morse index of a positive solution can be now defined in the classical way, and we will illustrate some qualitative consequences one should expect to deduce from such information. In particular, we show that zero Morse index (or more generally, non degenerate) solutions on the annulus are radial.

• **Serena Dipierro** - SISSA Trieste

*"Concentration of solutions for a singularly perturbed elliptic PDE problem in non-smooth domains"*

We consider the following singularly perturbed equation

$$-\epsilon^2 \Delta u + u = u^p \quad \text{in } \Omega,$$

where  $\Omega \subset \mathbb{R}^n$  is a bounded domain whose boundary has an  $(n-2)$ -dimensional smooth singularity. We study the problem both with Neumann and with mixed Dirichlet and Neumann boundary conditions. Assuming  $1 < p < (n+2)/(n-2)$ , we prove that, in both cases, concentration of solutions occurs at suitable points of the non smooth part of the boundary as the singular perturbation parameter tends to zero.

• **Alberto Ferrero** - Università degli Studi del Piemonte Orientale "Amedeo Avogadro"

*"On the spectrum of the fourth order Steklov eigenvalue problem"*

In my talk I would like to present some results obtained by me and other authors about the spectrum of the fourth order Steklov eigenvalue problem. It is well-known that the maximum/comparison principle for the biharmonic operator with homogeneous Dirichlet boundary conditions (b.c. in the sequel) is not in general satisfied; in this sense there are examples of domains with smooth boundary on which the maximum principle does

not hold true. On the other hand, it is quite easy to verify that the validity of the maximum principle for the biharmonic operator with homogeneous Navier b.c., is always satisfied in bounded domains with sufficiently smooth boundary. For this reason the authors of [BGM] tried to understand what happens when one considers conditions which are in some sense “intermediate” between Dirichlet and Navier b.c. In order to give an answer to this question they considered the so-called Steklov b.c. depending on a real parameter  $d$ . When studying the validity of the maximum principle with this kind of b.c., the Steklov spectrum naturally comes out, restricting the range of the admissible values of the parameter  $d$ . Subsequent papers were devoted to the study of the properties of this spectrum and of the corresponding eigenfunctions. We quote among the others the papers [AG], [BFG], [BG], [FGW], [GS]; some of them are essentially devoted to the shape optimization for the first eigenvalue of the Steklov problem seen as a function of the domain. My talk will be essentially focused on this last topic.

[AG] P.R.S. Antunes, F. Gazzola, Convex shape optimization for the least biharmonic Steklov eigenvalue, preprint

[BGM] E. Berchio, F. Gazzola, E. Mitidieri, Positivity preserving property for a class of biharmonic elliptic problems, J. Diff. Eq. 229, 2006, 1-23

[BFG] D. Bucur, A. Ferrero, F. Gazzola, On the first eigenvalue of a fourth order Steklov problem, Calculus of Variations 35, 2009, 103-131

[BG] D. Bucur, F. Gazzola, The first biharmonic Steklov eigenvalue: positivity preserving and shape optimization, Milan J. Math. 79, 2011, 247-258

[FGW] A. Ferrero, F. Gazzola, T. Weth, On a fourth order Steklov eigenvalue problem, Analysis 25, 2005, 315-332

[GS] F. Gazzola, G. Sweers, On positivity for the biharmonic operator under Steklov boundary conditions, Arch. Rat. Mech. Anal. 188, 2008, 399-427

• **Hans-Jürgen Freisinger** - Karlsruhe Institute of Technology (KIT)

*“An Interface Problem for the Nonlinear Schrödinger Equation”*

In this talk, we will consider the nonlinear Schrödinger equation  $-\Delta u + V(x)u = \Gamma(x)|u|^{p-1}u$  in  $\mathbb{R}^n$  where the coefficients  $V$  and  $\Gamma$  are composed of periodic functions  $V_1, V_2$  and  $\Gamma_1, \Gamma_2$  on the left and right of an interface plane.

We will discuss the existence of  $H^1$ -ground state solutions for this equation assuming that  $0 \notin \sigma(-\Delta + V)$ . Using constrained minimization on a generalized Nehari manifold, we derive an abstract existence criterion based on the ground state energies of the purely periodic problems with  $V \equiv V_1, \Gamma \equiv \Gamma_1$  and  $V \equiv V_2, \Gamma \equiv \Gamma_2$  and a more practical criterion based on the ground states themselves. We conclude with examples where these criteria are satisfied.

This is joint work with Wolfgang Reichel and based on a paper by Tomáš Dohnal, Michael Plum and Wolfgang Reichel.

• **Yuxin Ge** - Université Paris-Est Créteil Val de Marne

*“A new conformal invariant from generalized scalar curvature”*

In this talk we describe some new conformal invariants related to an inequality proved

recently by De Lellis and Topping. In particular, we prove that the De Lellis-Topping inequality is true on 3-dimensional and 4-dimensional Riemannian manifolds of nonnegative scalar curvature. More precisely, if  $(M^n, g)$  is a 3-dimensional or 4-dimensional closed Riemannian manifold with non-negative scalar curvature, then

$$\int_M \left| Ric - \frac{\bar{R}}{n} g \right|^2 dv(g) \leq \frac{n^2}{(n-2)^2} \int_M \left| Ric - \frac{R}{n} g \right|^2 dv(g),$$

where  $\bar{R} = vol(g)^{-1} \int_M R dv(g)$  is the average of the scalar curvature  $R$  of  $g$ . Equality holds if and only if  $(M, g)$  is Einstein manifold. We in fact study the following new conformal invariant

$$ds \tilde{Y}([g_0]) := \sup_{g \in \mathcal{C}_1([g_0])} \frac{ds vol(g) \int_M \sigma_2(g) dv(g)}{ds \left( \int_M \sigma_1(g) dv(g) \right)^2},$$

where  $\mathcal{C}_1([g_0]) := \{g = e^{-2u} g_0 \mid R > 0\}$ . By improving the analysis developed in the study of  $\sigma_k$ -Yamabe problem, we prove that  $\tilde{Y}([g_0]) \leq 1/3$  when  $n = 3$  and  $\tilde{Y}([g_0]) \leq 3/8$  when  $n = 4$ , which implies the above inequality. Some related results in high dimension  $n > 4$  are also described.

• **Sofia Giuffre'** - Mediterranea University of Reggio Calabria

*"Regularity theory of the gradient for general nonlinear parabolic systems"*

In this talk we deal with partial Hölder regularity of the spatial gradient of weak solutions to nonlinear parabolic systems of second order in divergence form of the following type:

$$(1) \quad - \sum_{i=1}^n D_i a^i(X, u, Du) + \frac{\partial u}{\partial t} = B^0(X, u, Du)$$

in the case of quadratic growth in the gradient.

• **Youssef Maliki** - University Aboubekr Belkaid of Tlemcen, Algeria

*"A multiplicity result for a Neumann problem on compact manifolds."*

We prove existence of two positive solutions of a Neumann problem on compact Riemannian manifold with boundary. The Neumann problem considered is the one which is equivalent to the problem of prescribing scalar and mean curvatures. We focus on the case of prescribed functions that change sign on the manifold.

• **Annalisa Massaccesi** Scuola Normale Superiore di Pisa

*"Currents with coefficients in a group and the Steiner Problem"*

in collaboration with Andrea Marchese

The Steiner problem consists in finding the shortest connected set including some fixed points; we show how this problem could be solved as a mass-minimizing problem for 1-dimensional rectifiable currents with coefficients in a suitable group. Thanks to the representation adopted for these currents, we can exploit the calibration method and we can analyse some examples.

• **Giampiero Palatucci** - Università degli Studi di Parma

*"A variational approach to subcritical problems involving the fractional Laplacian operator"*

Let  $\Omega$  be a bounded subset of  $\mathbb{R}^N$  and denote by  $S^*$  the optimal constant related to the critical Sobolev embedding  $H_0^s(\Omega) \hookrightarrow L^{2^*}(\Omega)$ , for any  $0 < s < N/2$ . In this talk, we present some variational techniques to investigate a natural approximation of  $S^*$  by subcritical embeddings. We show that for such approximations, optimal functions always exist and exhibit a concentration effect of the  $H^s$  energy at one point with an explicit concentration profile.

• **Cristina Pignotti** - Università di L'Aquila

*"A pointwise estimate for solutions of semilinear elliptic equations"*

We consider the problem  $\Delta u - W'(u) = 0$ , for  $x \in \Omega$ ,  $u = g$ , on  $\partial\Omega$ , where  $\Omega \subset \mathbb{R}^n$  is a Lipschitz bounded or unbounded domain,  $g : \partial\Omega \rightarrow \mathbb{R}$  is continuous and bounded and  $W : \mathbb{R} \rightarrow \mathbb{R}$  is a  $C^3$  potential with a unique global nondegenerate minimizer. We prove the existence of a solution satisfying a pointwise uniform exponential estimate.

• **Fabio Punzo** - Sapienza Università di Roma

*"Existence and nonexistence of patterns on Riemannian manifolds"*

The talk is concerned with existence and nonexistence of stable stationary nonconstant solutions (usually called patterns) to semilinear parabolic equations on compact Riemannian manifolds. In particular, a necessary and sufficient condition for existence of patterns on surfaces of revolution will be discussed. Such results have been obtained jointly with Catherine Bandle and Alberto Tesei.

• **Manel Sanchón** - Universitat de Barcelona

*"Regularity of stable solutions of  $p$ -Laplace equations through geometric Sobolev type inequalities"*

We prove a Sobolev and a Morrey type inequality involving the mean curvature and the tangential gradient with respect to the level sets of the function that appears in the inequalities. Then, as an application, we establish *a priori* estimates for semi-stable solutions of  $-\Delta_p u = g(u)$  in a smooth bounded domain  $\Omega \subset \mathbb{R}^n$ . In particular, we obtain new  $L^r$  and  $W^{1,r}$  bounds for the extremal solution  $u^*$  when the domain is strictly convex. More precisely, we prove that  $u^* \in L^\infty(\Omega)$  if  $n \leq p + 2$  and  $u^* \in L^{\frac{np}{n-p-2}}(\Omega) \cap W_0^{1,p}(\Omega)$  if  $n > p + 2$ .

• **Joaquim Serra** - Universitat Politècnica de Catalunya

*"Radial symmetry for semilinear equations via isoperimetric inequalities"*

We present a radial symmetry result for nonnegative solutions to the  $p$ -Laplace semilinear equation  $-\Delta_p u = f(u)$ , posed in a ball of  $\mathbb{R}^n$ . For the Laplacian ( $p = 2$ ) we obtain a new result that holds in every dimension  $n$  for certain positive discontinuous  $f$ . When  $p \geq n$  we prove radial symmetry for every nonnegative  $f$ , possibly discontinuous. We extend a method by P. L. Lions for the case  $p = n = 2$  which yields radial symmetry combining the

isoperimetric inequality and the Pohozaev identity. For variational anisotropic equations, similar arguments allow us to prove that level sets are Wulff shapes.

• **Raffaella Servadei** - Dipartimento di Matematica, Università della Calabria

*"Variational methods for equations driven by the fractional Laplacian"*

Motivated by the interest shown in the literature for non-local operators of elliptic type, in some recent joint papers with Enrico Valdinoci we have studied problems modeled by

$$\begin{cases} (-\Delta)^s u - \lambda u = |u|^{q-2}u & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega, \end{cases}$$

where  $s \in (0, 1)$  is fixed and  $(-\Delta)^s$  is the fractional Laplace operator,  $\Omega \subset \mathbb{R}^n$ ,  $n > 2s$ , is open, bounded and with Lipschitz boundary,  $\lambda > 0$ ,  $2^* = 2n/(n - 2s)$  is the fractional critical Sobolev exponent and  $2 < q \leq 2^*$ .

Aim of this talk will be to present some results which extend the validity of some existence theorems known in the classical subcritical and critical case of the Laplacian to the non-local framework.

In particular, in the critical setting (i.e.  $q = 2^*$ ) our theorems may be seen as the extension of the classical Brezis-Nirenberg result to the case of non-local fractional operators.

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